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# Galaxy And Mass Assembly (GAMA): bivariate functions of H $\alpha$ star-forming galaxies

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## ABSTRACT

We present bivariate luminosity and stellar mass functions of H $\alpha$  star-forming galaxies drawn from the Galaxy And Mass Assembly (GAMA) survey. While optically deep spectroscopic observations of GAMA over a wide sky area enable the detection of a large number of  $0.001 < \text{SFR}_{\text{H}\alpha} (\text{M}_{\odot} \text{ yr}^{-1}) < 100$  galaxies, the requirement for an H $\alpha$  detection in targets selected from an  $r$ -band magnitude-limited survey leads to an incompleteness due to missing optically faint star-forming galaxies. Using  $z < 0.1$  bivariate distributions as a reference we model the higher- $z$  distributions, thereby approximating a correction for the missing optically faint star-forming galaxies to the local star formation rate (SFR) and  $\mathcal{M}$  densities. Furthermore, we obtain the  $r$ -band luminosity functions (LFs) and stellar mass functions of H $\alpha$  star-forming galaxies from the bivariate LFs. As our sample is selected on the basis of detected H $\alpha$  emission, a direct tracer of ongoing star formation, this sample represents a true star-forming galaxy sample, and is drawn from both photometrically classified blue and red subpopulations, though mostly from the blue population. On average 20–30 per cent of red galaxies at all stellar masses are star forming, implying that these galaxies may be dusty star-forming systems.

**Key words:** surveys – galaxies: evolution – galaxies: luminosity function, mass function – galaxies: star formation.

## 1 INTRODUCTION

The observed univariate luminosity function (LF) is one of the fundamental measures of galaxy properties. It is usually one of the first results to be measured from galaxy surveys (e.g. Davis & Huchra 1982; Loveday et al. 1992, 2012; Lin et al. 1996; Norberg

et al. 2002; Blanton et al. 2003a). The importance of the LF, defined as the comoving source density with luminosity (or magnitude)  $L + \Delta L$ , extends to all areas of astronomy. In an observational context, it is used to quantify the mean space density of galaxies per unit luminosity and the evolution of statistical properties of a galaxy sample across cosmic time (e.g. Ly et al. 2007; Dale et al. 2010; Westra et al. 2010; Drake et al. 2013; Gunawardhana et al. 2013; Sobral et al. 2013, 2014). In theoretical modelling, the LF is a key ingredient needed to constrain the dark matter halo

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formation (e.g. Croton et al. 2006; Bower, McCarthy & Benson 2008). In an era of multiwavelength legacy surveys with intrinsically complicated multiband selections (e.g. Driver et al. 2011, Galaxy And Mass Assembly, GAMA, survey), understanding the effects of selection and systematic biases on the shape of the LF is imperative in obtaining reliable LF measurements.

Simply due to the existence of detection limits, no single survey can directly detect all sources to provide a complete and unbiased galaxy sample. The detection probability of an object is a function of a number of parameters, both external (e.g. survey selection, area and depth, star–galaxy separation, observing conditions and redshift, spectroscopic and target completeness) and intrinsic to the object (e.g. surface brightness, size and colour). As luminosity is strongly correlated with both sets of factors, the least luminous objects in any magnitude-limited survey have the poorest detection probabilities (Geller et al. 2012), thus occupying a relatively small volume. The low luminosity galaxies, although they do not dominate the luminosity budget of the Universe, greatly outnumber the luminous giants. As other studies have emphasized (Petrosian 1998), measuring the evolution of the slope of the faint end of the LF is a challenge. This arises because of the preferential bias against faint galaxies due to surface brightness limits (Sprayberry, Impey & Irwin 1996; Dalcanton et al. 1997; Geller et al. 2012), galaxy morphologies (Marzke et al. 1998; Tempel et al. 2011), spectral types (Folkes et al. 1999; Madgwick et al. 2002), environment (Xia et al. 2006; Tempel et al. 2009; Zandivarez & Martínez 2011) and colour (Blanton et al. 2001) as well as external issues (Driver et al. 2005; Loveday et al. 2012).

Any galaxy sample selected based on a parameter other than the primary survey selection criteria is biased as a result of the dual sample and survey selection. Gunawardhana et al. (2013) and Westra et al. (2010) present the  $H\alpha$  univariate LFs and determine the evolution of  $H\alpha$  star formation rate density (SFRD) in the local universe using  $H\alpha$  star-forming (SF) galaxy samples drawn, respectively, from the  $r$ -band magnitude-limited GAMA and Smithsonian Hectospec Lensing surveys. These studies show that their lowest redshift ( $z < 0.1$ ) samples are in fact the most complete and span the largest range in both intrinsic  $H\alpha$  luminosity ( $L_{H\alpha}$ ) and  $r$ -band absolute magnitude ( $M_r$ ), e.g. the GAMA  $z < 0.1$  sample probes  $30.5 \lesssim \log L_{H\alpha} (W) \lesssim 36$ ,  $-24 \lesssim M_r \lesssim -10$  (Gunawardhana et al. 2013) and  $7 \lesssim \log \mathcal{M}/M_\odot < 12$ , where  $\mathcal{M}$  is the galaxy stellar mass (Baldry et al. 2012). With increasing redshift, however, the sample completeness drops in the sense that a fraction of optically faint SF galaxies are missing from the higher- $z$  subsamples and this fraction increases with increasing redshift. As a consequence the final SFRDs based on bivariate selected samples are underestimated, manifesting as an apparent lack of evolution with redshift in contrast to current observations (Hopkins & Beacom 2006; Pérez-González et al. 2008; Karim et al. 2011; Sobral et al. 2014). In comparison, the LFs based on narrow-band surveys do not suffer from the same bias as their targets are selected using the quantity they aim to measure (see Gunawardhana et al. 2013, for a discussion on the advantages and disadvantages of broad-band and narrow-band surveys). In case of a bivariate selected sample, as is the case in this study, to recover the missing contribution from optically faint SF galaxies requires studying how the selection biases influence the LF.

The bivariate LF (Phillipps & Disney 1986) provides a powerful method of studying the luminosity density (LD) in different epochs inclusive of selection biases (e.g. bivariate brightness distributions, bivariate luminosity and size distribution). There is a rich collection of literature on using bivariate LFs to explore the space density of galaxies as a function of both survey selection

wavelength and surface brightness limits (e.g. Driver 1999; Blanton et al. 2001; Cross et al. 2001; Driver et al. 2005), galaxy size (e.g. Sodre & Lahav 1993; de Jong & Lacey 2000; de Jong et al. 2004; Cameron & Driver 2007), radio luminosity (e.g. Sadler, Jenkins & Kotanyi 1989; Ledlow & Owen 1996; Mauch & Sadler 2007), Sérsic index, stellar mass and spectral type (e.g. Ball et al. 2006), colour (e.g. Baldry et al. 2004) and in pairs of various galaxy properties (e.g. Blanton et al. 2003b; Driver et al. 2006) as well as bivariate ultraviolet/infrared LFs (e.g. Saunders et al. 1990; Takeuchi et al. 2013).

In this followup paper to Gunawardhana et al. (2013, hereafter Paper I), we explore the GAMA bivariate  $L_{H\alpha}$ – $M_r$  and  $L_{H\alpha}$ – $\mathcal{M}$  functions. Paper I presents the local star formation history as traced by  $H\alpha$  emitters contained within photometrically selected GAMA galaxies. The GAMA  $H\alpha$  LFs probe a wider range in luminosity than other results to-date and demonstrate a Gaussian-like drop in number density ( $\Phi$ ) at high luminosities, rather than the exponential drop characteristic of Schechter (1976) function. In Paper I we conclude that a Saunders et al. (1990) functional form, widely used to characterize radio and infrared LFs, is now also required to give a better description of  $H\alpha$  LFs. Despite the relatively large range in luminosity probed by the GAMA  $H\alpha$  LFs up to  $z < 0.34$ , the intrinsic SFRDs based on these LFs show a distinct lack of evolution in SFRD with increasing redshift. This apparent lack of evolution in SFRD is primarily caused by the sample selection. For GAMA, we find that the  $r$ -band apparent magnitude limit of the survey, along with the subsequent requirement for  $H\alpha$  detection, leads to an incompleteness due to missing bright  $H\alpha$  sources that are fainter than the  $r$ -band selection limit. While local estimates of SFRD measurements from a range of star formation rate (SFR)-sensitive wavelengths (e.g.  $H\alpha$ , [O II], [O III],  $H\beta$ ) based on narrow-band surveys and slitless spectroscopy data show an evolution with redshift (e.g. Jones & Bland-Hawthorn 2001; Shioya et al. 2008; Sobral et al. 2013), those based on broad-band surveys are almost always underestimated (e.g. Westra et al. 2010; Paper I) as a consequence of the bivariate sample selection. The primary aims of this investigation are to: (a) model the low redshift bivariate function to use as a reference to account for the missing optically faint SF galaxies at higher- $z$  ( $z < 0.34$ ), (b) measure the (moderate) redshift evolution of stellar mass and SFRDs, (c) characterize the univariate  $M_r$  LFs and stellar mass functions (SMFs) of  $H\alpha$  SF galaxies and compare the results with LFs and SMFs of photometrically classified blue galaxies, and (d) explore the characteristics of photometrically classified blue and red SF subpopulations in GAMA.

The layout of this paper is as follows. We briefly describe the GAMA survey, our sample selection criteria and selection biases in Section 2. In Section 3, we describe the derivation of the bivariate functions, taking into account different survey selection criteria. The resulting bivariate functions are presented in Sections 4 and 6. These sections also include the univariate functions obtained from integrating the bivariate functions over  $L_{H\alpha}$  axis. Finally, in Section 5 we give detail of the functional forms used to fit the bivariate functions, and in Sections 5.2 and 6.3, we infer SFR and  $\mathcal{M}$  densities by integrating the best-fitting functional forms to the bivariate functions. Our results are discussed in Section 7 and we conclude in Section 8.

The assumed cosmological parameters are  $H_0 = 70 \text{ km s}^{-1} \text{ Mpc}^{-1}$ ,  $\Omega_M = 0.3$ , and  $\Omega_\Lambda = 0.7$ . All magnitudes are presented in the AB system. A Chabrier (2003) initial mass function (IMF), commonly used to derive stellar masses in the literature, is used to derive the stellar mass measurements used in this study (Taylor et al. 2011) and the same IMF used

in Paper I (i.e. Baldry & Glazebrook 2003, IMF) is used to estimate SFRs. As these two IMF forms are sufficiently similar, no significant systematic effect is introduced by adopting them. To avoid confusion we state in the figure caption which IMF is used to obtain the results shown.

## 2 THE GAMA SURVEY AND BIVARIATE SAMPLE SELECTION

Our study utilizes data from the GAMA<sup>1</sup> survey (Baldry et al. 2010; Robotham et al. 2010; Driver et al. 2011). GAMA is a spectroscopic survey undertaken at the Anglo Australian Telescope with 2-degree Field fibre feed and the AAOmega multi-object spectrograph. AAOmega provides a resolution of 3.2 Å full width at half-maximum with complete spectral coverage from 3700 to 8900 Å (Sharp et al. 2006; Driver et al. 2011; Hopkins et al. 2013). GAMA-I covers three equatorial fields at approximately 09, 12 and 15h, hereafter G09, G12 and G15, of 48 deg<sup>2</sup> each. G09 and G15 are limited to a depth of  $r_{AB} < 19.4$  while G12 extends to  $r_{AB} < 19.8$ .

For the current analysis, we use GAMA-I spectroscopic data consisting of GAMA, Sloan Digital Sky Survey (SDSS; Adelman-McCarthy et al. 2008), 2-degree Field Galaxy Redshift Survey (2dFGRS; Colless et al. 2001) and Millennium Galaxy Catalogue (Driver et al. 2005) sources. A detailed description of the data, the sample selection and the measurement of physical properties of galaxies such as SFRs is given in section 2 of Paper I. Briefly, the emission-line measurements used for this investigation are measured from each flux calibrated spectrum, assuming a single Gaussian profile and a common redshift and line width within an adjacent set of lines (e.g. H $\alpha$ , the [N II]  $\lambda\lambda$  6548, 6583 and [S II] doublets), and simultaneously fitting the continuum local to the set of lines. This method of measuring fluxes does not take into account the effects of the underlying stellar absorption on Balmer line fluxes. To correct for this effect, we apply a constant correction to H $\alpha$  and H $\beta$  fluxes following the prescription of Hopkins et al. (2003). A comprehensive discussion of GAMA spectroscopic measurement process is presented in Hopkins et al. (2013), and further analyses on the effects of the assumption of a constant stellar absorption correction are presented in Paper I and in Gunawardhana et al. (2011).

The derivation of stellar absorption, aperture and dust obscuration corrected H $\alpha$  luminosities ( $L_{H\alpha}$ ) is described in section 3 of Paper I. The stellar masses ( $\mathcal{M}$ ) and absolute magnitudes  $k$ -corrected to  $z = 0$  ( $M_r$ ) have been derived using the stellar template spectrum that best fits  $u, g, i, r, z$  GAMA photometry (StellarMasses v08; Taylor et al. 2011). We have not attempted to correct the rest-frame  $r$ -band continuum flux for H $\alpha$  emission contamination as the contamination is <2 per cent for over 98 per cent of the lowest redshift data (H $\alpha$  spectral line is redshifted out of the  $r$ -band filter at  $z \sim 0.05$ ), and does not change the results we present in this analysis. Additionally, we compute absolute magnitudes based on SDSS Petrosian  $r$ -band magnitudes at  $z = 0.1$ , hereafter  $M_r^{0.1}$  (KCORRECT v4\_2; Blanton & Roweis 2007) to compare our results with previous GAMA studies (e.g. Baldry et al. 2012; Loveday et al. 2012).

We use the same sample of galaxies as in Paper I. Briefly, our sample includes all emission-line galaxies with H $\alpha$  fluxes ( $F_{H\alpha}$ ) greater than a detection limit of  $25 \times 10^{-20}$  W m<sup>-2</sup>, and H $\alpha$  emission signal-to-noise ratio greater than 3 that are classified as SF based on the prescription of Kewley & Dopita (2002). The emission-line

measurements for galaxies in the three GAMA regions that have been observed previously in earlier spectroscopic campaigns (e.g. SDSS, 2dFGRS, 6dFGRS) are either taken from their respective survey data bases or accounted for through spectroscopic incompleteness corrections if the respective spectra are not flux calibrated (e.g. 2dFGRS spectra). The sample properties and trends with redshift are explored in sections 2 and 3 of Paper I. To summarize, the sample used for the calculation of  $f_{V, H\alpha}$  includes the following galaxies.

(i) GAMA observations with observed H $\alpha$  emission above the detection limit of  $25 \times 10^{-20}$  W m<sup>-2</sup>. AGNs are removed from the sample using the standard optical emission-line ([N II]  $\lambda$ 6584/H $\alpha$  and [O III]  $\lambda$ 5007/H $\beta$ ) diagnostics (Baldwin, Phillips & Terlevich 1981, hereafter BPT) diagnostics and the Kewley & Dopita (2002) prescription. The two line classifications (Brough et al. 2011; Paper I) are used to recognize AGNs if one of the four spectral line measurements required for a BPT diagnostics is unavailable. A small subset of GAMA galaxies where none of these methods can be employed is included in the sample, as these are more likely to be SF than AGNs (Cid Fernandes et al. 2011).

(ii) SDSS observations with H $\alpha$  emission and continuum signal-to-noise ratio greater than 3. The AGNs are removed as described above using SDSS line measurements available from MPA-JHU data base.<sup>2</sup>

Figs 1 and 2 demonstrate the limitations of a bivariate selected sample. Fig. 1 shows the  $M_r$  distributions of all GAMA galaxies in four different redshift ranges compared to the distributions of galaxies with reliable H $\alpha$  detections (i.e.  $F_{H\alpha} > 25 \times 10^{-20}$  W m<sup>-2</sup>) and galaxies with  $F_{H\alpha} > 1 \times 10^{-18}$  W m<sup>-2</sup> (i.e. the flux limit used in the  $V_{\max}$  calculations; section 4 of Paper I). The distributions of blue and red subpopulations, as defined in equation (4) of this paper and in Loveday et al. (2012), of galaxies within each distribution are shown in the insets. The distributions of H $\alpha$  detected galaxies at each redshift range comprise both blue and red galaxies, though blue galaxies dominate the distributions. Also, the  $z < 0.1$  and  $0.1 < z < 0.15$  distributions of the H $\alpha$  detected galaxies show a clear blue–red bimodal distribution, while the lack of such a trend in the higher redshift distributions is likely partly due to the difficulty in reliably measuring the H $\alpha$  feature in low signal-to-noise ratio weak-line systems at higher redshifts, and partly due to the smaller luminosity range probed at higher redshifts. Even though the distribution of H $\alpha$  detected sample is bimodal, the sample, after imposing an H $\alpha$  flux limit for the estimation of  $V_{\max}$  (Paper I), is biased against red weak-line galaxies at all redshifts, which are likely to be dusty SF galaxies with low signal-to-noise ratio spectra (Section 6.1).

Fig. 2 further demonstrates how the bivariate selection acts to limit the range of  $L_{H\alpha}$  and  $M_r$  probed by the LFs with increasing redshift.

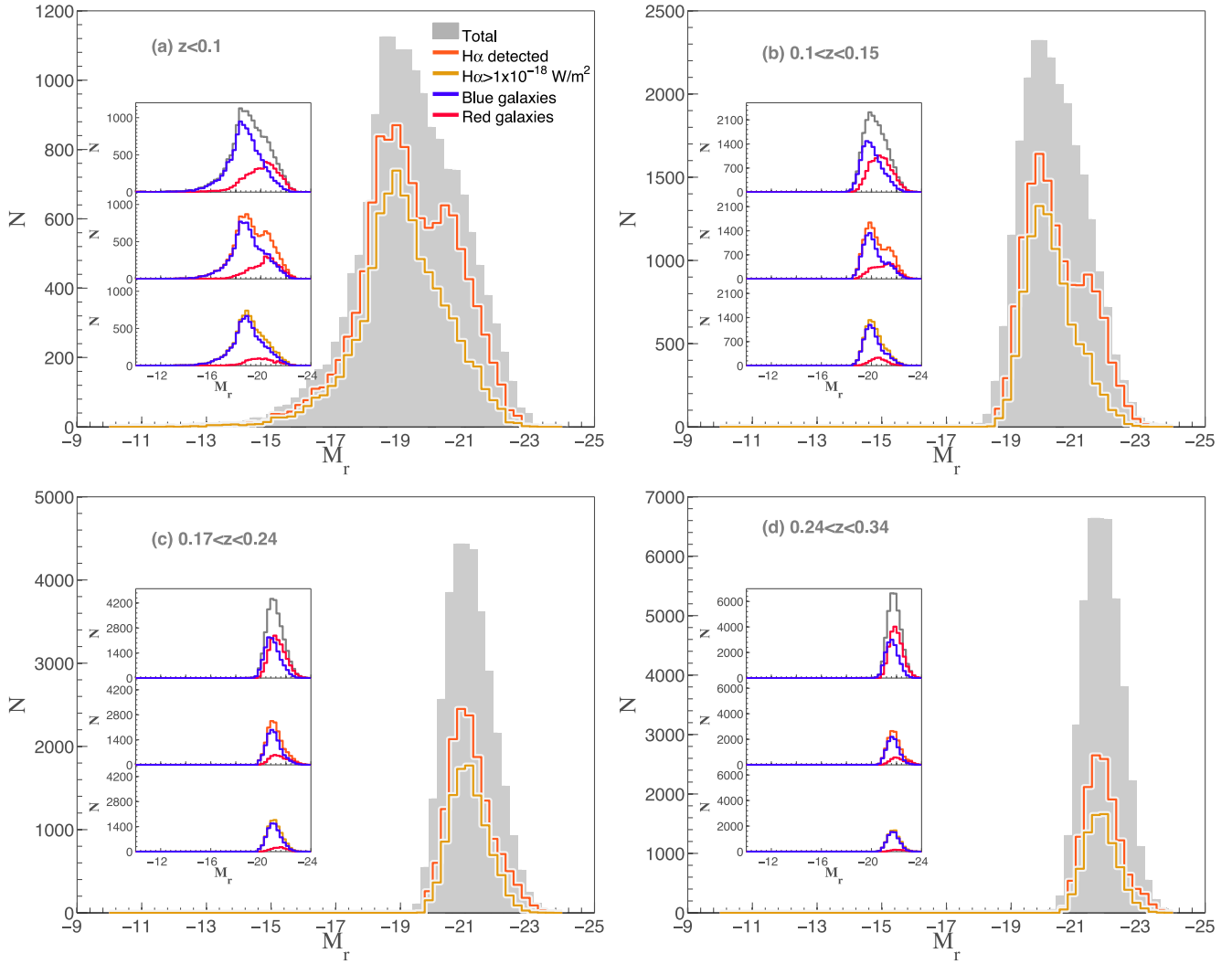
The  $z < 0.1$  sample probes the largest range in both  $M_r$  and  $L_{H\alpha}$ . The dual H $\alpha$ – $M_r$  selection prevents the optically faint SF galaxies from entering the sample at all redshifts, with its effects becoming more significant with increasing redshift. In order to assess the extent of this effect, we now consider the bivariate LFs.

## 3 CONSTRUCTING BIVARIATE FUNCTIONS

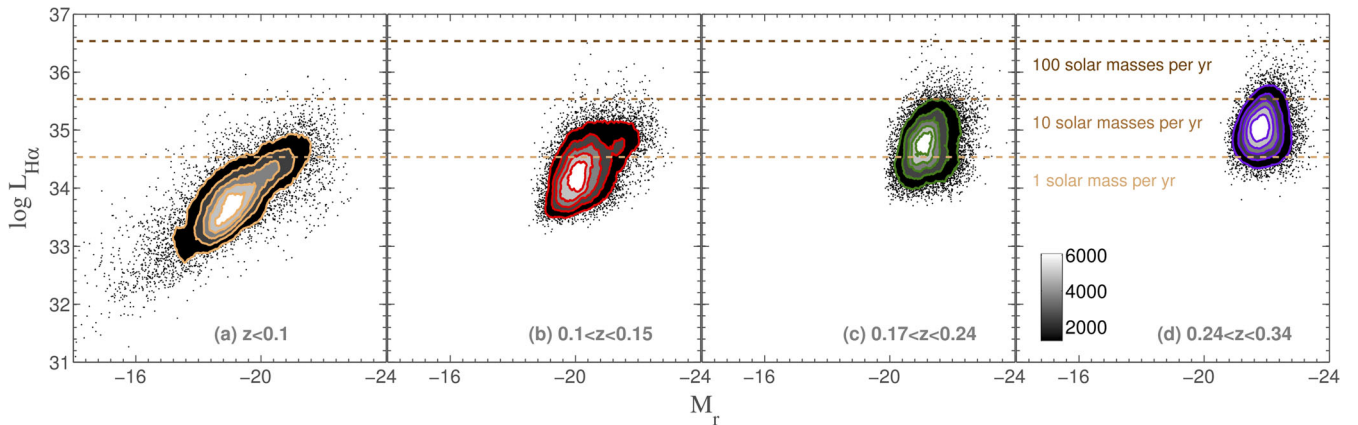
We use three different LF estimators (the classical  $1/V_{\max}$  method, the density corrected  $1/V_{\max}$  method and the bivariate stepwise

<sup>1</sup> <http://www.gama-survey.org/>

<sup>2</sup> [http://www.mpa-garching.mpg.de/SDSS/DR7/raw\\_data.html](http://www.mpa-garching.mpg.de/SDSS/DR7/raw_data.html)



**Figure 1.** The  $M_r$  distribution of GAMA galaxies split into four redshift bins. Each panel shows three distributions: all galaxies (grey histogram), those with observed  $H\alpha$  flux greater than the detection limit of  $25 \times 10^{-20} \text{ W m}^{-2}$  (orange histogram), and those with  $H\alpha$  fluxes  $> 1 \times 10^{-18} \text{ W m}^{-2}$ , i.e. the flux limit (yellow histogram). Each of the three histograms are further divided to indicate the distributions of galaxies classified as blue and red based on equation (4). These are shown within the three insets in each main panel. The three insets top-to-bottom show the distributions of blue and red subpopulations corresponding to all galaxies (top), galaxies with  $H\alpha$  fluxes  $> 25 \times 10^{-20} \text{ W m}^{-2}$  (middle) and galaxies with  $H\alpha$  fluxes  $> 1 \times 10^{-18} \text{ W m}^{-2}$  (bottom).



**Figure 2.** The bivariate  $\log L_{H\alpha}$  (W) and  $M_r$  distributions split into four redshift bins. The dual  $r$ -band apparent magnitude and  $H\alpha$  flux selection of our sample led to an incompleteness in optically faint SF galaxies. The  $z < 0.1$  sample probes the largest range in  $H\alpha$  luminosity and  $M_r$ , therefore it is the most complete. The range probed by the higher redshift samples progressively drops with redshift.



maximum likelihood, SWML) to derive the bivariate LFs presented in Section 4. The formulation of each of the three methods is described below.

### 3.1 The ‘classical’ method

The  $1/V_{\max}$  technique (Schmidt 1968), also referred to as the ‘classical method’, is widely used to estimate the comoving space density as a function of luminosity.

The formulation of the  $1/V_{\max}$  method inclusive of incompleteness corrections for univariate LFs is described in section 4 of Paper I. The bivariate function derived using this definition is

$$\Phi[\log L_{H\alpha}, M_r] \times \Delta \log L \Delta M_r = \sum_i \frac{1}{V_{i,\max}}. \quad (1)$$

In this equation,  $V_{i,\max}$  represents the maximum volume out to which the  $i$ th object would be visible and still be part of the survey,  $\Delta \log L$  and  $\Delta M_r$  define the luminosity and magnitude bin widths, respectively.

The GAMA-I sample used in this study is subjected to two different  $r$ -band magnitude limits ( $r < 19.4$  for G09, 15 and  $r < 19.8$  for G12; Driver et al. 2011) and an emission-line selection. Given these constraints, the definition of  $V_{i,\max}$  is

$$V_{i,\max} = \min[(V_{i,\max,H\alpha}), (V_{i,\max,r}), (V_{i,z_{\lim}})] \times c_i, \quad (2)$$

where  $V_{i,\max}$  is the minimum volume that a galaxy  $i$  would have given the maximum volumes for that galaxy computed using an  $H\alpha$  flux limit of  $1 \times 10^{-18} \text{ W m}^{-2}$  ( $V_{i,\max,H\alpha}$ ) and the magnitude limit of the GAMA survey ( $V_{i,\max,r}$ ), and the volume ( $V_{i,z_{\lim}}$ ) of the redshift slice ( $z_{\lim}$ ) that  $i$ th galaxy resides in. The completeness correction,  $c_i$ , takes into account both the imaging and spectroscopic incompleteness. The estimation of  $c_i$  is described in detail in Paper I and in Loveday et al. (2012).

The classical method has the advantages of simplicity and it gives simultaneously both the shape and the absolute normalization of the LF, however, it can be susceptible to cosmic (sample) variance (Efsthathiou, Ellis & Peterson 1988; Willmer 1997; Baldry et al. 2012) when the survey volume is small. For this reason, large data sets covering a substantial portion of the sky are generally required to avoid the shape of the LF being distorted due to large-scale structure.

There are more sophisticated methods of estimating LFs to account for these disadvantages. Two such methods are the density corrected  $1/V_{\max}$  corrections (Baldry et al. 2006; Cole 2011; Mahtessian 2011; Baldry et al. 2012) and the SWML (Efsthathiou et al. 1988), a variant of the method proposed by Sandage, Tammann & Yahil (1979).

Finally, the univariate LF is obtained by integrating equation (1) over one of the variables.

### 3.2 Density corrected maximum volume corrections

Baldry et al. (2006, but also see Mahtessian 2011; Baldry et al. 2012) describe a modification to the  $1/V_{\max}$  technique that takes into account the radial variation in the large-scale structure. They define a maximum volume weighted by density,  $V'_{\max}$ ,

$$V'_{i,\max} = \frac{\rho_{\text{ddp}}(z_{\min}; z_{\max,i})}{\rho_{\text{ddp,mean}}(z_1; z_2)} \times V_{i,\max} = f_V \times V_{i,\max}, \quad (3)$$

where  $\rho_{\text{ddp}}(z_1; z_2)$  is the number density of a density defining population (DDP) between redshifts  $z_1$  and  $z_2$ . DDP is a volume-limited sample and  $z_1, z_2$  are the minimum and maximum redshifts

of that volume-limited sample (Baldry et al. 2006, 2012).  $\rho_{\text{ddp}}(z_1; z_2)$  is estimated separately for each GAMA field, and the average between the three fields is taken to be  $\rho_{\text{ddp,mean}}$ .

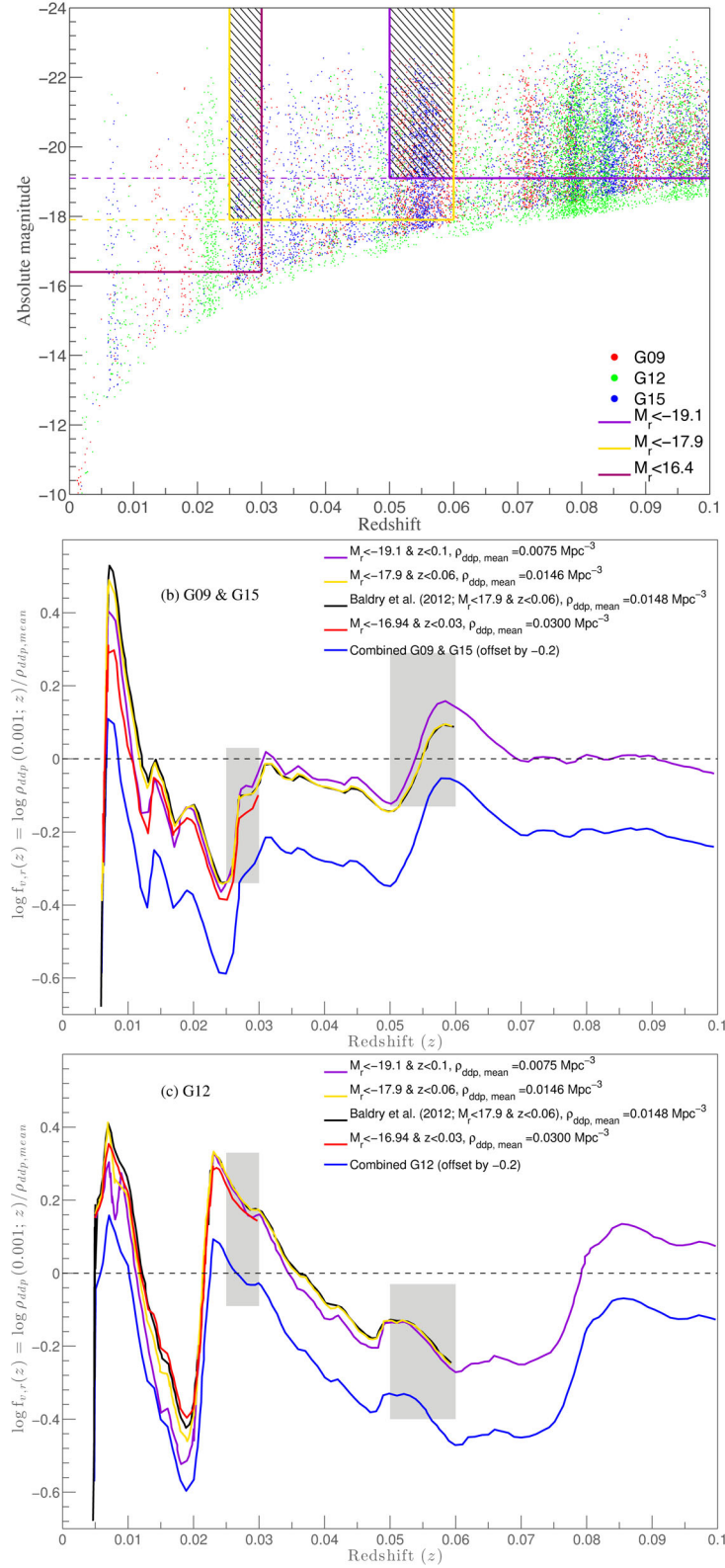
As we investigate the evolution of the bivariate  $L_{H\alpha}$ – $M_r$  function over a moderate range in redshift, the density weights ( $f_v$ ) for  $V_{\max,r\text{-band}}$  and  $V_{\max,H\alpha}$  are estimated separately using several overlapping volume-limited samples to improve the accuracy of  $V'_{\max,i}$  (Mahtessian 2011). The GAMA sample selection criteria detailed in Baldry et al. (2012), not restricted to their redshift range, is adopted to estimate density weights for  $V_{\max,r\text{-band}}$ . We use  $f_{v,r}$  to denote the density weights estimated using this sample, which has a univariate  $r$ -band apparent magnitude selection. Fig. 3(a) shows the volume-limited sample definitions used in the calculation of  $f_{v,r}$  for the  $z < 0.1$  sample. The variation in  $f_{v,r}$  is quantified separately for the G09 and G15 (Fig. 3 b), and G12 (Fig. 3 c) fields because of their different magnitude limits. In the regions where two volume-limited samples are allowed to overlap, shown as hatched and shaded regions in Fig. 3, the  $f_{v,r}$  corrections are combined such that the transition from one volume-limited sample to the other is smooth. The blue line in Figs 3(b) and (c) show the final  $f_{v,r}$  versus redshift relation used in the calculation of  $V'_{\max,r}$  offset by  $-0.2$  in log space for legibility. Also shown for comparison is the  $f_{v,r}$  relation from Baldry et al. (2012) for  $z < 0.06$  GAMA galaxies (black line). The negligible difference between the blue and black lines is a result of the different  $r$ -band magnitude types used as inputs to KCORRECT v4.2 (Blanton & Roweis 2007).

As the galaxy sample used for this study has a dual  $r$ -band magnitude and  $H\alpha$  flux selection, we also explored the impact of estimating the density corrections using only the  $H\alpha$  detections in a  $r$ -band selected sample. We use  $f_{v,H\alpha}$  to denote this correction. This serves, in principle, as a different correction,  $f_{V,H\alpha}$ , for  $V_{\max,H\alpha}$ , although it should be similar in practice. To estimate  $f_{V,H\alpha}$ , we add 2dFGRS observations<sup>3</sup> with  $\eta$ , a measure of the average absorption/emission-line strength of a galaxy that strongly correlates with  $H\alpha$  equivalent width, greater than  $-1.2$  (Madgwick et al. 2002) to the SF galaxy sample used for this study (Section 2). The AGNs are removed as described above using the 2dFGRS spectral line catalogue (Lewis et al. 2002).

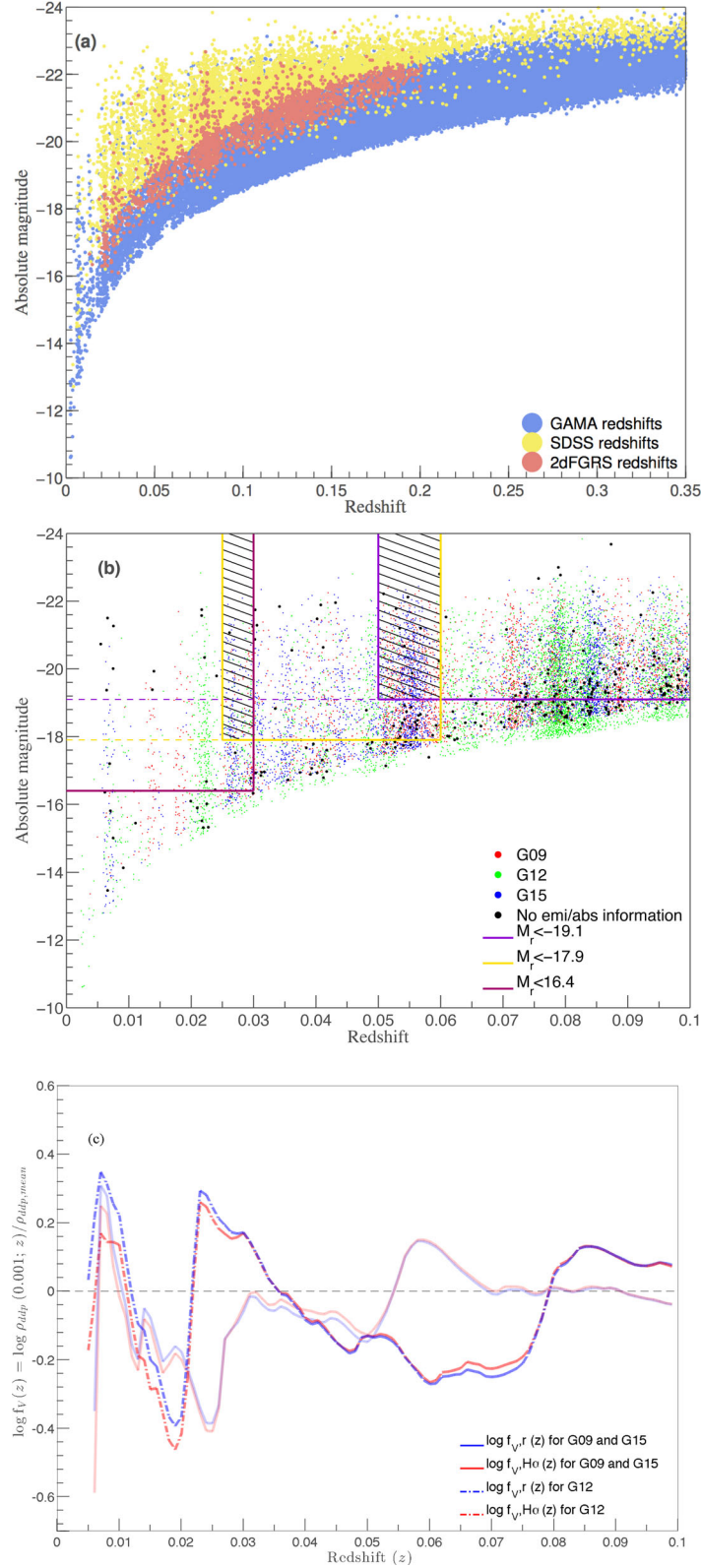
The distribution of  $r$ -band absolute magnitudes in redshift for the selected GAMA  $H\alpha$  emission-line galaxies with spectra originating from GAMA, SDSS and 2dFGRS surveys is shown in Fig. 4(a), with the GAMA survey providing the deepest spectroscopic observations followed by 2dFGRS and SDSS surveys. The same volume-limited sample definitions introduced in Fig. 3 are used to calculate  $f_{v,H\alpha}$ , but from an  $r$ -band magnitude versus redshift distribution comprising only  $H\alpha$  SFR galaxies (Fig. 4 b). The black points in Fig. 4(b) indicate galaxies with spectra originating from a survey other than GAMA, 2dFGRS or SDSS for which we currently do not have spectral line information, and are not included in the sample used to determine  $f_{v,H\alpha}$ . The spectroscopic incompleteness arising from the exclusion of these objects leads to a small discrepancy between  $f_{v,r}$  and  $f_{v,H\alpha}$  as demonstrated in Fig. 4(c). Even though this discrepancy is largest at the lowest redshifts, as might be expected due to the relatively low number of galaxies sampled by the lowest redshift volume-limited sample in comparison to other volume-limited samples, it has a negligible effect on the density corrected bivariate functions.

An alternative approach that does not require binning was developed by Sandage et al. (1979, hereafter STY). The STY maximum

<sup>3</sup> [www2.aao.gov.au/2dfgrs/](http://www2.aao.gov.au/2dfgrs/)



**Figure 3.** (a) The distribution of absolute  $r$ -band magnitudes in redshift for *all*  $z < 0.1$  galaxies colour coded by their GAMA region identifier. Using this distribution we define three overlapping volume-limited samples, which are then used in the derivation of  $\log f_{v,r}$  as a function of redshift for each volume-limited sample (b and c). As the three GAMA fields have two different magnitude limits,  $f_{v,r}$  is derived separately for G09 and G15 ( $r < 19.4$ ), and G12 ( $r < 19.8$ ). The blue line, offset from the rest for legibility, indicates the variation in final  $f_{v,r}$  used for the analysis. This line traces the  $f_{v,r}$  versus redshift relation estimated from the lowest redshift volume sample (i.e. red line) until  $z \sim 0.03$  (i.e. first shaded region corresponding to the hatched region in the top panel) before smoothly transitioning to the relation derived from the second volume-limited sample (yellow line) and so on. The  $f_{v,r}$  versus redshift relations derived in Baldry et al. (2012) for  $z < 0.06$  galaxies are shown for comparison.



**Figure 4.** (a) The distribution of absolute  $r$ -band magnitudes of  $\text{H}\alpha$  SFR galaxies in redshift, colour coded by the survey from which the spectra are taken. (b) Same as Fig. 3, but for SF galaxies. The black points indicate the absolute magnitudes of galaxies for which we do not have any spectra. (c) The  $f_{v,\text{H}\alpha}$  (red) and  $f_{v,r}$  (blue) versus redshift relations for G09 and G15 (solid line), and G12 (dashed line). The lack of spectral line information makes the derived  $f_{v,\text{H}\alpha}$  a lower limit, however, we demonstrate that the final  $f_{v,\text{H}\alpha}$  versus redshift relation is close to the true relation (i.e. the true relation here is that of  $f_{v,r}$ ) as there are only a few galaxies in a given redshift slice without this information. Note that as we use all galaxies to derive  $f_{v,r}$ , the lack of spectral line measurements does not affect  $f_{v,r}$  estimates.



likelihood LF estimator, although not biased by the presence of the large-scale structure, requires the assumption of a parametric form for the LF. A ‘non-parametric’ variant of STY method called the SWML estimator was introduced by Efstathiou et al. (1988), mainly to overcome the inconvenience of not being able to adequately establish whether the chosen parametrization represents a good fit to the data (Efstathiou et al. 1988; Willmer 1997). While this technique is insensitive to density fluctuations (Sandage et al. 1979; Efstathiou et al. 1988; Willmer 1997), the luminosity bins in SWML methods are highly correlated such that any issue that occurs in a given bin may affect the whole LF. In Appendix A, we describe the formulation of the SWML estimator for bivariate functions. A comparison between the lowest redshift bivariate  $L_{H\alpha}$ – $M_r$  function constructed using SWML method and that constructed using the density corrected  $V_{\max}$  method is presented in Section 3.3.

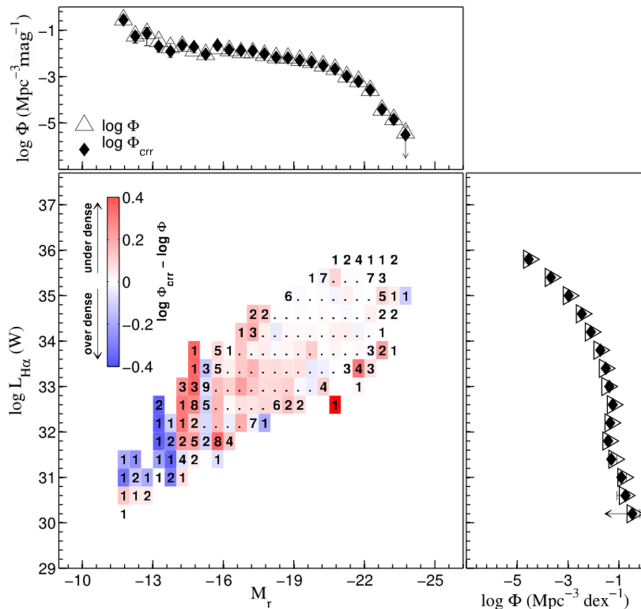
### 3.3 A comparison of bivariate LF estimators

The formulation of the classical  $1/V_{\max}$  and the density corrected  $1/V_{\max}$  is described in Sections 3.1 and 3.2.

We present the residual between the two bivariate functions. We focus here on the  $z < 0.1$  slice as this is the one which is most likely to differ between the two methods due to the size of the volume surveyed.

Fig. 5 shows the residual maps obtained by subtracting the bivariate  $L_{H\alpha}$ – $M_r$  function derived using the density corrected  $1/V_{\max}$  method from that derived using the classical  $1/V_{\max}$  method.

As mentioned above, the bivariate functions based on the classical  $1/V_{\max}$  method, especially the  $\Phi$  estimates at faint  $M_r$  and  $L_{H\alpha}$ , are affected by large-scale structure. The density corrected  $1/V_{\max}$  technique is designed to account for radial variations through  $\rho_{\text{ddp}}$  defined in Section 3.2. This is demonstrated in Fig. 5, where the redder colour indicates that the applied density weight



**Figure 5.** The residual map showing the difference in number densities derived using the classical ( $\Phi$ ) and the density corrected ( $\Phi_{\text{corr}}$ )  $V_{\max}$  methods for the  $z < 0.1$  bivariate functions. The colours indicate the difference between  $\Phi_{\text{corr}}$  and  $\Phi$ . A positive difference (redder colours) implies that  $f_v$  corrects  $V_{\max}$  for an underdensity and negative (bluer colours) means the correction is for an overdensity. Top and right-hand panels show the  $M_r$  and  $H\alpha$  univariate LFs.

corrects the classical  $1/V_{\max}$  for an underdensity and vice versa, with the colour intensity showing the significance of that correction for each  $L_{H\alpha}$ – $M_r$  bin. As expected the density correction becomes progressively more important towards fainter  $M_r$  and  $L_{H\alpha}$  values (i.e. towards increasingly smaller volumes). The top and right-hand panels of Fig. 5 show the  $M_r$  and  $H\alpha$  LFs obtained from integrating the bivariate  $L_{H\alpha}$ – $M_r$  functions based on the classical (open symbols) and density corrected (filled symbols)  $1/V_{\max}$  methods in  $L_{H\alpha}$  and  $M_r$  directions, respectively. Clearly, the density corrections to the bivariate functions have a small effect, and are limited to the faintest end of the bivariate function. This is not surprising given the low volume being probed combined with the small numbers of galaxies contributing to each bin. Higher- $z$  residual maps are devoid of significant differences in this comparison. We note that we see an almost identical result if we use the  $f_{V,r}$  rather than  $f_{V,H\alpha}$  in making the density-dependent correction to the  $V_{\max,H\alpha}$ . In summary, we see a marginal difference in the bivariate functions between  $1/V_{\max}$  and the density corrected  $1/V_{\max}$ , while no statistical difference is observed in the univariate LFs.

A similar conclusion is reached from the comparison between SWML with the density corrected  $V_{\max}$  (Appendix A).

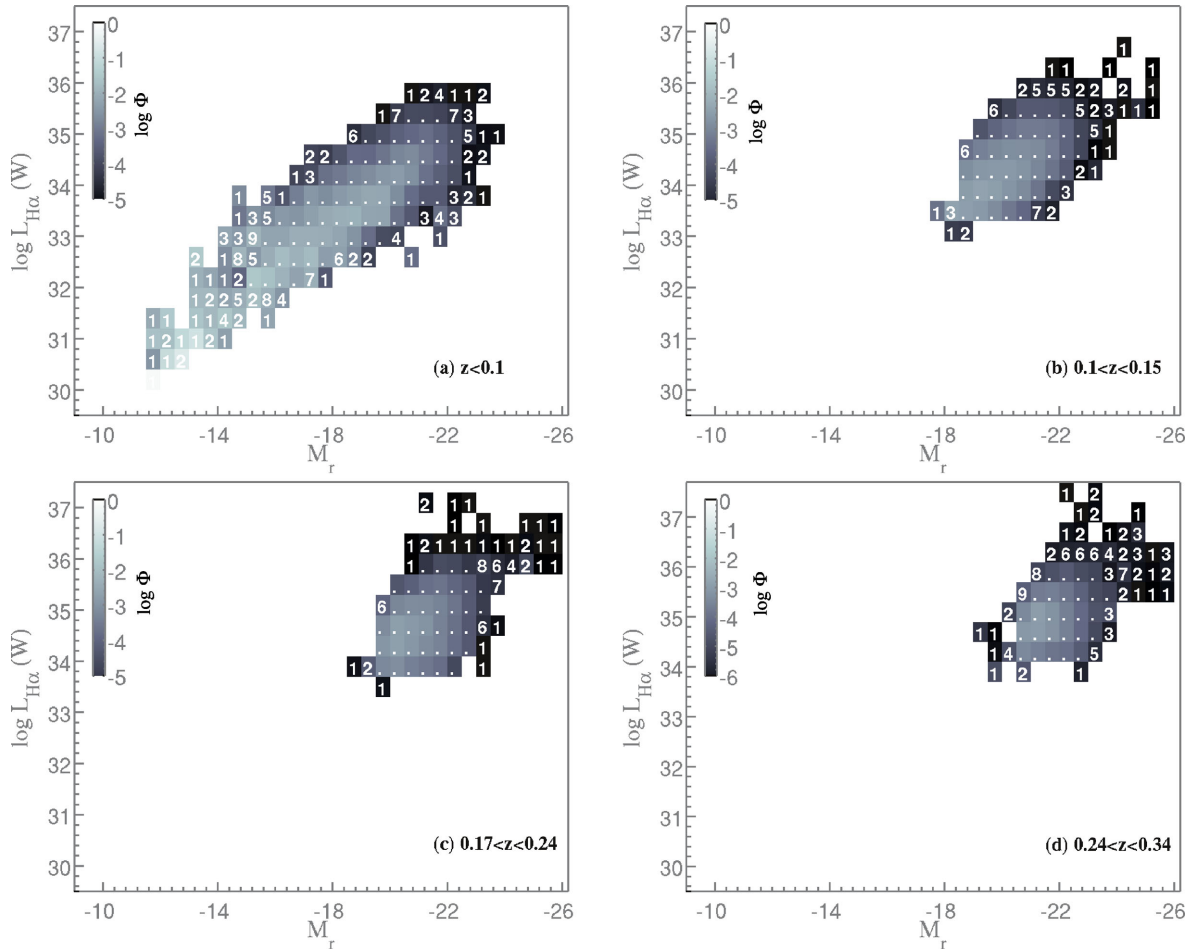
### 4 BIVARIATE $L_{H\alpha}$ – $M_r$ FUNCTIONS

The GAMA bivariate  $L_{H\alpha}$ – $M_r$  functions in redshift bins constructed using the classical  $V_{\max}$  method are shown in Fig. 6. Due to the magnitude-limited nature of the survey, the range in  $M_r$  and  $L_{H\alpha}$  probed by the bivariate functions progressively decreases with increasing redshift. Particularly, the lowest- $z$  bivariate LF width with respect to  $M_r$  and  $L_{H\alpha}$  (i.e. the horizontal and vertical lengths of the bivariate distribution as a function of  $M_r$  or  $L_{H\alpha}$ , respectively) indicates an overall decrease towards fainter  $L_{H\alpha}$ – $M_r$ . This is supported by the reduction in number towards fainter  $M_r$  and  $L_{H\alpha}$ . The decrease in horizontal width with increasing  $L_{H\alpha}$  is likely to be a result of our sample being biased against red star formers (Section 2). The range in  $L_{H\alpha}$ – $M_r$  probed by the three higher- $z$  bivariate functions become more limited and faint  $L_{H\alpha}$ – $M_r$  bins become more incomplete with increasing redshift. The  $L_{H\alpha}$ – $M_r$  bins with low galaxy number statistics are indicated in Fig. 6. Note that the errors in  $\log \Phi$  for  $L_{H\alpha}$ – $M_r$  bins with small numbers of galaxies (e.g. numbers  $\lesssim 3$ ) is large.

We further assign a photometric blue or red class to each SF galaxy in our sample. For the analysis presented in this section, the blue–red classification is determined using the  $g - r$  and  $M_r^{0.1}$  colour–magnitude cut defined in Zehavi et al. (2011) and used by Loveday et al. (2012)

$$(g - r)_{\text{model}}^{0.1} = 0.15 - 0.03 M_r^{0.1}. \quad (4)$$

There are a number of blue–red galaxy classification schemes in the literature (e.g. Bell et al. 2003; Baldry et al. 2004; Peng et al. 2010; Taylor et al. 2015). Baldry et al. (2004, 2012), for example, advocate a non-linear cut in  $(u - r)$  rest-frame colour and  $M_r$ . Since the observed bivariate data distribution in colour–magnitude plane is non-linear (Baldry et al. 2004), a non-linear cut in  $(u - r)$ – $M_r$  space would improve our blue–red selection. As our goal in this part of the analysis is to compare the univariate functions computed from the bivariate functions with the univariate functions of Loveday et al. (2012), the same colour–magnitude selection employed by Loveday et al. (2012) is used. A comprehensive discussion of different colour–magnitude classifications and their implications is presented in Taylor et al. (submitted).



**Figure 6.** Bivariate  $L_{\text{H}\alpha}$ – $M_r$  functions of the  $\text{H}\alpha$  SF galaxies computed using the classical  $1/V_{\text{max}}$  method split into four redshift bins with redshift increasing across. The  $L_{\text{H}\alpha}$ – $M_r$  bins with less than 10 galaxies in them have their number of galaxies quoted, while the dots indicate bins with 10 or more galaxies. The grey-scale corresponds to the number density in the unit of  $\text{Mpc}^{-3} \text{dex}^{-1} \text{mag}^{-1}$ .

The bivariate  $L_{\text{H}\alpha}$ – $M_r$  functions of blue and red  $\text{H}\alpha$  SF subpopulations split into four redshift bins are shown in Fig. 7. The range in  $M_r$  and  $L_{\text{H}\alpha}$  probed by the blue  $L_{\text{H}\alpha}$ – $M_r$  functions and their number densities are similar to that of the total  $\text{H}\alpha$  SF LFs (Fig. 6) at all redshifts, implying that the SF bivariate LF, while drawn from both photometrically classified blue and red subpopulations, consists mostly of blue galaxies.

#### 4.1 $r$ -band galaxy LFs of $\text{H}\alpha$ star formers

By integrating the bivariate  $L_{\text{H}\alpha}$ – $M_r$  functions over the  $L_{\text{H}\alpha}$  axis, the  $M_r$  LFs of the total  $\text{H}\alpha$  SF sample and photometrically classified blue and red  $\text{H}\alpha$  SF subsamples can be recovered. We explore the evolution of LDs in Section 5.2.

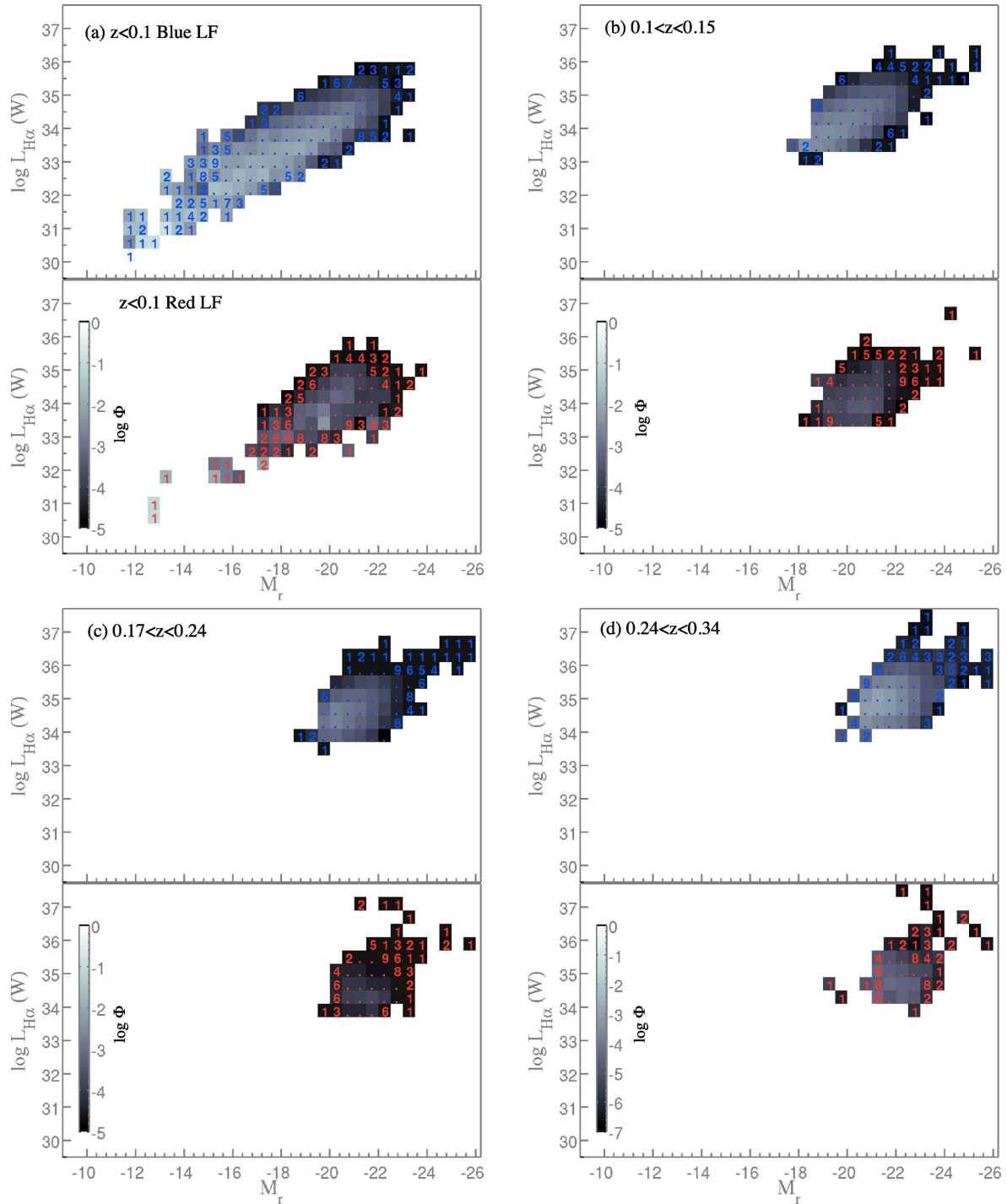
Fig. 8 shows the  $z < 0.1$   $r$ -band galaxy LFs of all  $\text{H}\alpha$  SF (left-hand panel), blue  $\text{H}\alpha$  SF (right top panel) and red  $\text{H}\alpha$  SF (right bottom) galaxies computed from integrating the corresponding bivariate functions.<sup>4</sup> These LFs are compared with the GAMA  $r$ -

band LFs from Loveday et al. (2012) and SDSS  $r$ -band LFs from Blanton et al. (2005), which are shown as open squares and diamonds, respectively, in Fig. 8.

The  $z < 0.1$   $M_r^{0.1}$  LF of all  $\text{H}\alpha$  SF galaxies (left-hand panel of Fig. 8) closely follows the  $M_r$  LF of all GAMA galaxies from Loveday et al. (2012). In fact, the bright end of the all  $\text{H}\alpha$  SF  $M_r^{0.1}$  LF very closely tracks the GAMA red  $M_r^{0.1}$  LF (Loveday et al. 2012), demonstrating that a significant fraction of the  $z < 0.1$  galaxies classified as red have detected  $\text{H}\alpha$  emission at all  $M_r^{0.1}$  values. This is further corroborated by the colour–magnitude distributions (Fig. 8 inset) of all  $z < 0.1$  galaxies regardless of star formation (filled contours) and  $z < 0.1$   $\text{H}\alpha$  SF galaxies (purple contours), where the purple contours extend beyond the blue population. The small discrepancy evident between the blue  $M_r^{0.1}$  LF of  $\text{H}\alpha$  SF galaxies and that from Loveday et al. (2012) is likely a result of the differences in the formulation of the  $1/V_{\text{max}}$  technique. By taking into account the minimum of  $V_{\text{max},r}$  and  $V_{\text{max},\text{H}\alpha}$  (Paper I) rather than only  $V_{\text{max},r}$  as done by Loveday et al. (2012), where we take into account the maximum volume corrections based on both  $r$ -band magnitude and  $\text{H}\alpha$  flux limits.

Furthermore, we use the Kauffmann et al. (2003) criteria on discriminating pure-SF and SF–AGN composites to further remove likely SF–AGN composites from the red SF sample. The resultant  $M_r^{0.1}$  LF is shown as open red diamonds in the right bottom panel of Fig. 8. The open symbols are slightly displaced from the  $M_r^{0.1}$

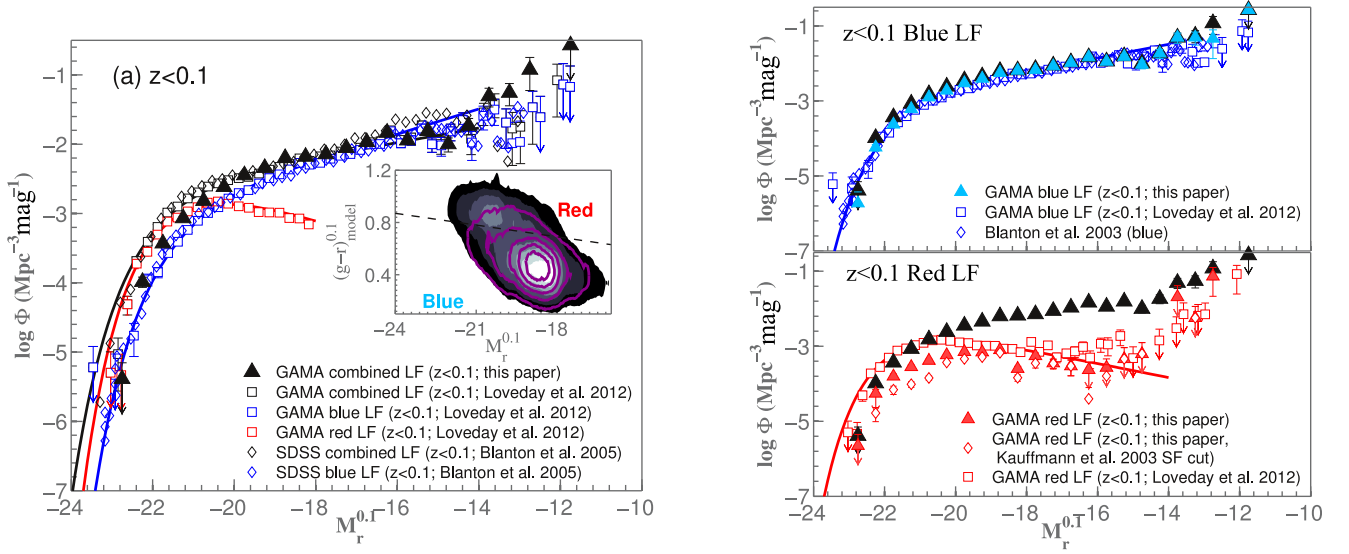
<sup>4</sup> The bivariate LFs shown in Figs 6 and 7 use  $r$ -band absolute magnitude  $k$ -corrected to  $z = 0$ . To obtain the  $M_r^{0.1}$  LFs shown in Fig. 8, we calculate the bivariate LFs using  $r$ -band absolute magnitude  $k$ -corrected to  $z = 0.1$  to compare our results directly with Loveday et al. (2012).



**Figure 7.** Bivariate  $L_{\text{H}\alpha}$ – $M_r$  functions of blue and red  $\text{H}\alpha$  SF subpopulations constructed using the classical  $V_{\text{max}}$  method split in four redshift bins. The grey-scale shown along side each red bivariate LF indicates the number densities ( $\Phi$ ) in the unit of  $\text{Mpc}^{-3} \text{dex}^{-1} \text{mag}^{-1}$  for both blue and red functions of the respective redshift range. Like in Fig. 6, the  $L_{\text{H}\alpha}$ – $M_r$  bins with less than 10 galaxies in have their number of galaxies indicated.

LF constructed using SF galaxies selected based on the Kewley & Dopita (2002) criteria (filled symbols) at the bright end of the LF, and overlap with the filled symbols at the faint end, implying that only a small fraction of red galaxies are removed from the original red SF sample by the Kauffmann et al. (2003) cut. We have also used the method of Cid Fernandes et al. (2011) for differentiating SF galaxies from AGNs. Even with the AGN/SF cuts recommended by Cid Fernandes et al. (2011), more than 50 per cent of the red galaxy

sample at a given redshift retain their SF status. So that a fraction of galaxies classified as red at all  $M_r^{0.1}$  values are in fact currently forming stars. Moreover, the analysis of Lara-López et al. (2013) exploring the properties of SDSS and GAMA galaxies find that a large fraction of galaxies detected in GAMA are SF in comparison to SDSS as GAMA is deeper than SDSS, and therefore more sensitive to low-mass galaxies at low redshift, which are mostly dominated by star formation.



**Figure 8.** GAMA  $M_r$  total (left-hand panel), blue (right top) and red (right bottom) LFs of SF galaxies derived from integrating the  $z < 0.1$  bivariate  $L_{H\alpha}-M_r$  functions. These LFs are compared against the  $M_r$  LFs presented in Loveday et al. (2012) for all GAMA galaxies (open squares) and Blanton et al. (2005) for all SDSS galaxies (open diamonds). The solid lines show the best-fitting Schechter functions to  $z < 0.1$  LFs from Loveday et al. (2012). The left-hand panel inset shows the data density distributions in linear space of all  $z < 0.1$  galaxies regardless star formation (filled contours) and SF galaxies with  $F_{H\alpha} > 1 \times 10^{-18} \text{ W m}^{-2}$  (solid contours) in  $(g-r)_{\text{model}}^{0.1}$  and  $M_r^{0.1}$  plane. The colour-magnitude cut (equation 4) used to split blue and red galaxies is shown as a dashed line. The open red diamonds in right bottom panel indicate the red SF  $M_r^{0.1}$  LF constructed using the (pure) SF galaxies selected according to Kauffmann et al. (2003) SF/AGN prescription instead of that of Kewley & Dopita (2002).

The higher- $z$  ( $z$  up to 0.34)  $M_r^{0.1}$  LFs of all  $H\alpha$  SF galaxies (Fig. B2) overlap with the GAMA blue LFs of Loveday et al. (2012) over similar redshift ranges, while the fractional contribution from red  $H\alpha$  SF galaxies to the GAMA red  $M_r^{0.1}$  LFs (Loveday et al. 2012) progressively drops with increasing redshift. The contours in the insets of Fig. 9 depicting the distribution of all galaxies (filled contours) and SF galaxies (magenta contours) at different redshifts show that the contours of SF galaxies become more restricted to the blue subpopulation with increasing redshift, collaborating the drop in red SF number densities seen in Fig. B2. This is most likely due to the flux limit biasing our sample against these systems (Fig. 1), and possibly also due to the difficulty in measuring spectral lines in more obscured, weak emission-line systems at higher redshifts.

## 5 MODELLING OF BIVARIATE $L_{H\alpha}$ -DEPENDENT FUNCTIONS AND IMPLICATIONS FOR LUMINOSITY AND SFRDS

LFs, emission line and photometric alike, are traditionally parametrized with a Schechter (1976) function, which is then integrated to obtain a luminosity or SFRD. The linear form of the Schechter function,<sup>5</sup>

$$\Phi(L)dL = \Phi^* \left(\frac{L}{L^*}\right)^\alpha \exp\left(-\frac{L}{L^*}\right) d\left(\frac{L}{L^*}\right), \quad (5)$$

<sup>5</sup> The logarithmic form of the Schechter function is expressed as

$$\Phi(\log L)d \log L = \ln(10)\Phi^*10^{(\log L - \log L^*)(\alpha+1)} \exp[-10(\log L - \log L^*)]d \log L.$$

behaves as a power law with a slope  $\alpha$  for luminosities ( $L$ ) less than the characteristic luminosity ( $L^*$ ) and as an exponential for  $L > L^*$ , with the normalization given by  $\Phi^*$ . From this, the predicted LD is given by

$$\rho_{\text{Lfit}} = \int_0^\infty L \Phi(L) dL = \Phi^* L^* \Gamma(\alpha + 2), \quad (6)$$

where  $\Gamma$  is the Gamma function, and a conservative limit is given by

$$\rho_{\text{Lfit,lim}} = \int_0^{L_{\min}} L \Phi(L) dL = \Phi^* L^* \Gamma(\alpha + 2, L_{\min}/L^*), \quad (7)$$

where  $\Gamma$  is now the incomplete Gamma function.

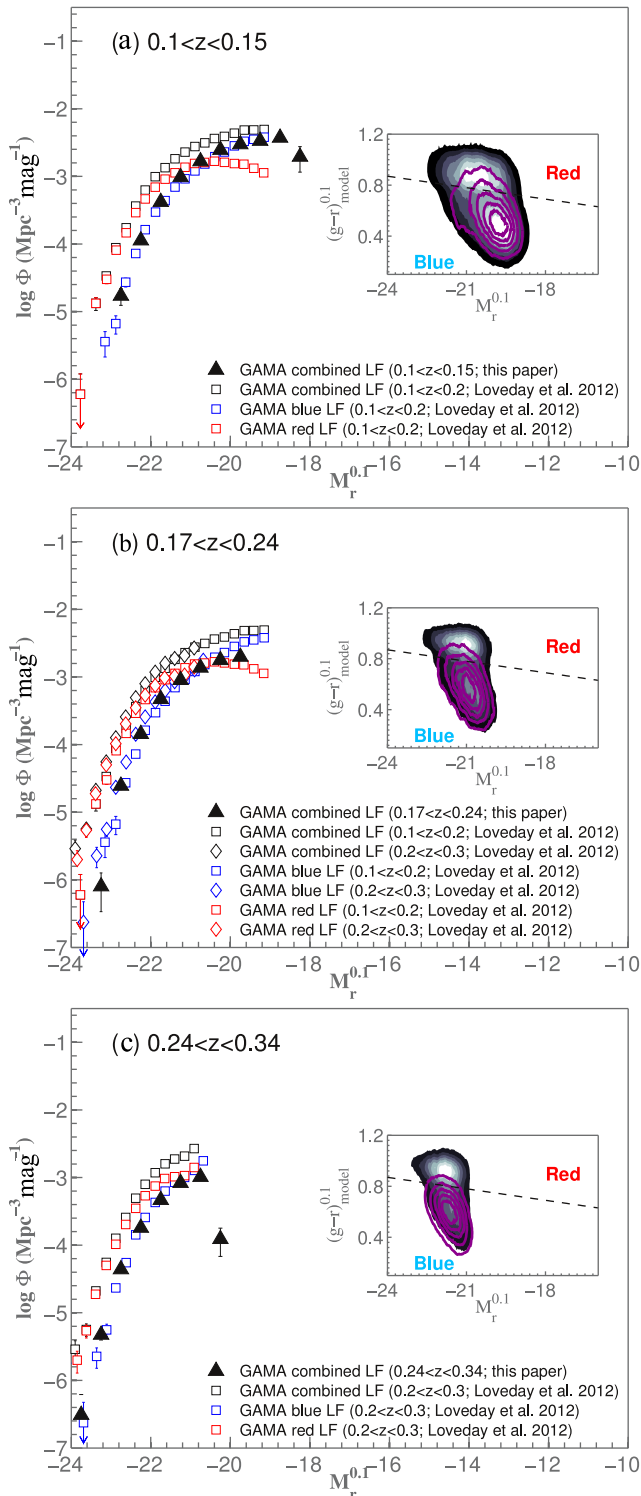
Galaxy broad-band LFs are, generally, well described by a Schechter function (e.g. Hill et al. 2010; Loveday et al. 2012). Several studies (e.g. Blanton et al. 2005), however, find that a double Schechter function is best suited at capturing the whole shape of the galaxy LF, especially if the range in magnitude probed is relatively large.

While the galaxy broad-band LFs show an exponential-like fall in  $\Phi$  with increasing brightness, the SF LFs (e.g.  $H\alpha$ , far-infrared, radio) show a less steep fall in  $\Phi$  (Saunders et al. 1990; Salim & Lee 2012; Paper I). For this reason, to characterize the SF LFs we adopt a Saunders et al. (1990) function. The linear form of the Saunders function

$$\Phi(L)dL = \Phi^* \left(\frac{L}{L^*}\right)^\alpha \exp\left[-\frac{1}{2\sigma^2} \log^2\left(1 + \frac{L}{L^*}\right)\right] d\left(\frac{L}{L^*}\right), \quad (8)$$

behaves in a similar fashion to a Schechter function for  $L < L^*$  and as a Gaussian in log luminosity with a width ( $\sigma$ ) for  $L > L^*$ .





**Figure 9.** GAMA higher- $z$   $M_r^{0.1}$  LFs of all SF galaxies computed by integrating the bivariate  $L_{H\alpha}$ - $M_r$  functions. The open symbols in each panel show the GAMA all, blue and red  $M_r^{0.1}$  LFs of Loveday et al. (2012) over similar redshift ranges. The insets show the data density distributions in  $(g-r)_{\text{model}}^{0.1}$  and  $M_r^{0.1}$  of all galaxies regardless of star formation (grey shading) and SF galaxies with  $F_{H\alpha} > 1 \times 10^{-18} \text{ W m}^{-2}$  (magenta contours). These distributions correspond to the redshift ranges given in the plot key. The colour-magnitude cut (equation 4) is shown as a dashed line.

We fit Schechter functions to the  $M_r^{0.1}$  LFs of SF galaxies shown in Figs 8 and B2 using a Levenberg–Marquardt routine to find the minimum  $\chi^2$  to the binned LF data points. The resultant Schechter functional fits and their best-fitting parameters are presented in Fig. 10 and Table 1. Due to the lack of faint SF galaxies at higher- $z$ , a consequence of the survey magnitude selection, the best-fitting slopes for the  $0.1 < z < 0.15$ ,  $0.17 < z < 0.24$  and  $0.24 < z < 0.34$  LFs cannot be measured reliably from the observed LF. Instead, we constrain the faint-end slopes of higher- $z$  LFs to be equal to the best-fitting slopes of their respective  $z < 0.1$  LFs. The low- $z$  red SF galaxy LF has a poorly constrained  $\alpha$  due to a lack of bright galaxies necessary to disentangle the degeneracy between  $\alpha$  and  $M^*$ , therefore we assume  $\alpha = -1.29$  (i.e.  $\alpha$  estimated from the  $z < 0.1$   $M_r^{0.1}$  LF all  $H\alpha$  SF galaxies) for the higher- $z$  red LFs.

### 5.1 Functional fits to bivariate functions

In this section, we explore a simple analytic approach to modelling the bivariate  $L_{H\alpha}$ - $M_r$  function. Such a parametrization of bivariate functions is useful in comparing the distributions drawn from differently selected samples and for studying the redshift evolution inclusive of selection biases.

Choloniewski (1985) and de Jong & Lacey (2000) developed a formalism to link the distribution of galaxy scale sizes, assumed to be Gaussian, to the luminosity parametrized by a Schechter function. Their analytic expression as well as other related functional forms have widely been used to model bivariate brightness profiles (e.g. de Jong & Lacey 2000; Cross et al. 2001; Driver et al. 2005; Ball et al. 2006), size-luminosity (e.g. de Jong & Lacey 2000; Huang et al. 2013) and colour-luminosity relationships (e.g. Chapman et al. 2003; Chapin, Hughes & Aretxaga 2009). Similar functional forms have also been formulated by Yang et al. (2005) and Cooray & Milosavljević (2005) in the context of conditional LFs that specify the average number of galaxies with luminosities in the range  $L \pm \Delta L/2$  that reside in a given halo mass.

Our motivation for modelling the GAMA bivariate functions is to correct for the apparent lack of evolution in  $H\alpha$  SFRDs with redshift reported in Paper I. The low-redshift bivariate model can be used as reference to gain an understanding of the extent to which the higher- $z$  ( $0.1 \lesssim z \lesssim 0.34$ ) samples are affected by incompleteness.

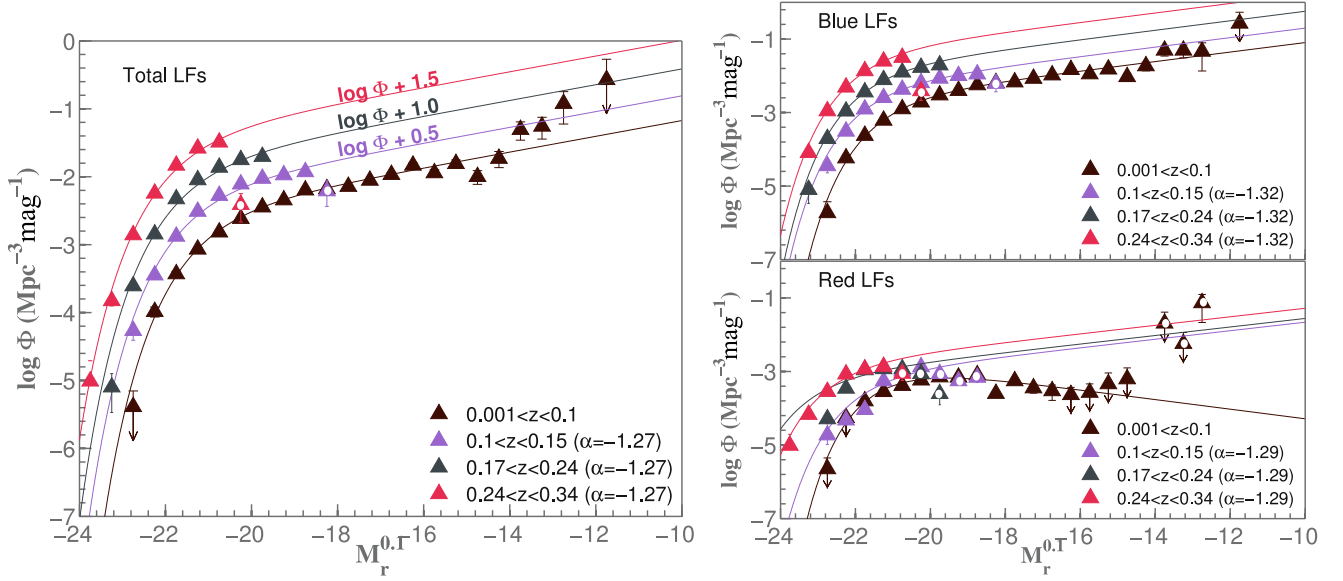
Below we describe the construction of a simple analytic model, denoted  $\Psi(M, L_{H\alpha})$ , to describe the bivariate functions presented in this paper, assuming that the bivariate functions can be written as a product of two functions (Choloniewski 1985; Corbelli, Salpeter & Dickey 1991; Hopkins 1998). Naturally, as a Schechter function best describes the  $M_r$  LFs presented in Section 4.1 and a Saunders function best describes the GAMA  $H\alpha$  LFs (Paper I), these functions are adopted to represent the bivariate distributions.

To link these functional forms in a bivariate analytic relation, we begin by establishing how  $L_{H\alpha}^*$  in the  $H\alpha$  LF varies as a function of  $M_r$ . The relationship between  $L_{H\alpha}^*$  and  $M_r$  is clearly evident in Fig. 11, where we divide the lowest- $z$   $L_{H\alpha}$ - $M_r$  function by  $L_{H\alpha}$  LF at a given  $M_r$ . The symbols indicate the best-fitting  $L_{H\alpha}^*$  in different magnitude ranges, estimated by fitting a Saunders function (equation 8) with a non-varying faint-end slope (blue) to the data and fitting a Gaussian (green),

$$\Phi(L)dL = \frac{\Phi^*}{\sigma\sqrt{2\pi}} \exp \left[ -\frac{1}{2\sigma^2} \log^2 \left( \frac{L}{L^*} \right) \right] d \left( \frac{L}{L^*} \right), \quad (9)$$

with a Gaussian width,  $\sigma$ , that is allowed to vary to the data.





**Figure 10.** The best-fitting Schechter functions to the  $M_r^{0.1}$  LFs shown in Figs 8, 9 and B2. The faint-end slope,  $\alpha$ , is fixed at  $-1.29$  for the higher- $z$  combined (left-hand panel) and red SF (right bottom) LFs. It is fixed at  $-1.32$  for the higher- $z$  blue LFs (right top). Each higher- $z$  LF is scaled up by the factors stated in the left-hand panel to aid legibility.

**Table 1.** The best-fitting Schechter function parameters corresponding to the functional fits shown in Fig. 10, and the LDs computed from integrating the fits.

$z$	$M_r^*$	$\log \Phi^*$ ( $\text{Mpc}^{-3}$ )	$\alpha$	$\log \rho_{\text{Lfit}}$ ( $L_\odot \text{Mpc}^{-3}$ )
Combined				
$z < 0.10$	$-20.93 \pm 0.15$	$-2.41 \pm 0.14$	$-1.29 \pm 0.06$	$7.97 \pm 0.05$
$0.10 < z < 0.15$	$-21.21 \pm 0.06$	$-2.54 \pm 0.05$	$-1.29^a$	$7.96 \pm 0.03$
$0.17 < z < 0.24$	$-21.34 \pm 0.13$	$-2.64 \pm 0.15$	$-1.29^a$	$7.91 \pm 0.09$
$0.24 < z < 0.34$	$-21.52 \pm 0.02$	$-2.72 \pm 0.04$	$-1.29^a$	$7.90 \pm 0.03$
Blue				
$z < 0.10$	$-20.80 \pm 0.07$	$-2.46 \pm 0.07$	$-1.32 \pm 0.05$	$7.89 \pm 0.01$
$0.10 < z < 0.15$	$-21.21 \pm 0.08$	$-2.62 \pm 0.06$	$-1.32^b$	$7.89 \pm 0.06$
$0.17 < z < 0.24$	$-21.25 \pm 0.07$	$-2.66 \pm 0.11$	$-1.32^b$	$7.86 \pm 0.08$
$0.24 < z < 0.34$	$-21.40 \pm 0.06$	$-2.71 \pm 0.08$	$-1.32^b$	$7.89 \pm 0.05$
Red				
$z < 0.10$	$-20.76 \pm 0.09$	$-2.85 \pm 0.08$	$-0.67 \pm 0.07$	$7.31 \pm 0.04$
$0.10 < z < 0.15$	$-21.44 \pm 0.25$	$-3.45 \pm 0.17$	$-1.29^a$	$7.14 \pm 0.06$
$0.17 < z < 0.24$	$-22.74 \pm 1.07$	$-4.00 \pm 0.27$	$-1.29^a$	$7.12 \pm 0.16$
$0.24 < z < 0.34$	$-22.16 \pm 0.17$	$-4.16 \pm 0.22$	$-1.29^a$	$6.82 \pm 0.14$

<sup>a</sup> $\alpha$  is fixed at  $-1.29$ , the best-fitting slope for the  $z < 0.1$  blue and red combined  $M_r^{0.1}$  LF of H $\alpha$  SF galaxies.

<sup>b</sup> $\alpha$  is fixed at  $-1.32$ , the best-fitting slope for the  $z < 0.1$  blue  $M_r^{0.1}$  LF of H $\alpha$  SF galaxies.

The relationship between  $L_{\text{H}\alpha}^*$  and  $M_r$  seen in Fig. 11 can be approximated as

$$L_{\text{H}\alpha}^* = L_{\text{H}\alpha}^*(M_r^0) 10^{-0.4\beta(M_r - M_r^0)}. \quad (10)$$

Using equation (10) to connect equations (5) and (8), and expressing the distribution in terms of  $M_r$ , we arrive at the full bivariate expression:

$$\Psi(M_r, L_{\text{H}\alpha}) = \Phi(M_r) \times \Phi(M_r, L_{\text{H}\alpha}) = 0.4 \ln(10)$$

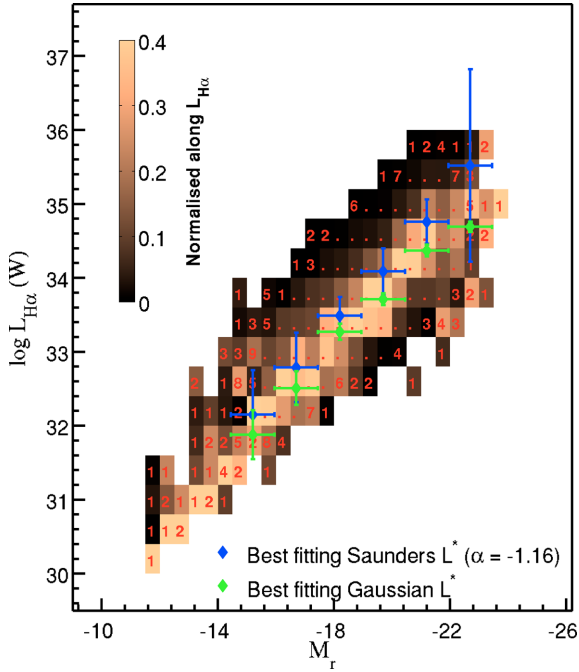
$$\Psi^* 10^{-0.4(\alpha_M + 1)(M_r - M_r^*)}$$

$$\times \exp \left[ -10^{-0.4(M_r - M_r^*)} \right]$$

$$\times \left\{ \left[ \frac{L_{\text{H}\alpha}}{L_{\text{H}\alpha}^*(M_r^0)} \right] \left[ 10^{0.4\beta(M_r - M_r^0)} \right] \right\}^{\alpha_L}$$

$$\times \exp \left\{ -\frac{1}{2\sigma^2} \log^2 \left[ 1 + \left( \frac{L_{\text{H}\alpha}}{L_{\text{H}\alpha}^*(M_r^0)} \right) \left( 10^{0.4\beta(M_r - M_r^0)} \right) \right] \right\}. \quad (11)$$

We assume  $M_r^0 = -19.0$  and fit the, now, seven free parameter model to the  $z < 0.1$   $L_{\text{H}\alpha}$ - $M_r$  function through a non-linear  $\chi^2$  minimization routine based on Levenberg-Marquardt method using the Poisson errors of the bivariate function measurements. The resultant best-fitting model is shown in Fig. 12 and the best-fitting model parameters are given in Table 2.



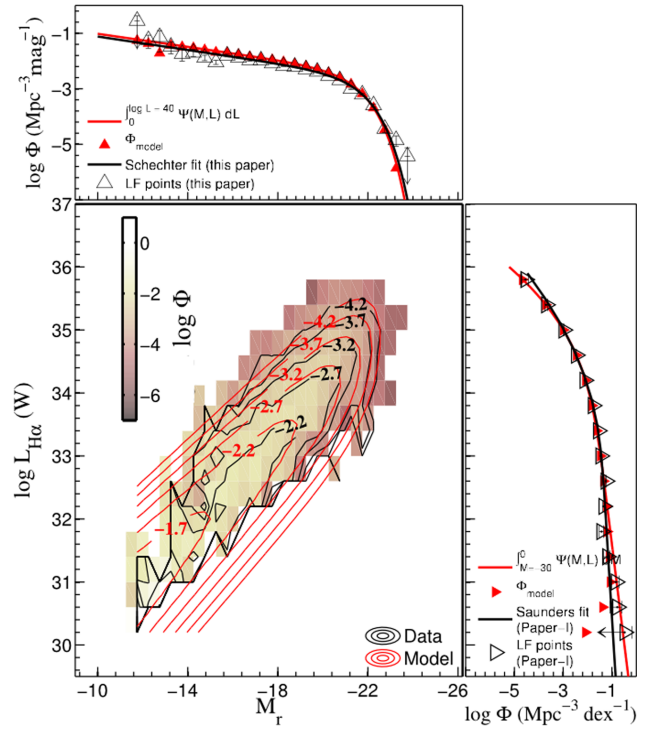
**Figure 11.** The low- $z$  bivariate function (Fig. 6 a) normalized along the  $L_{\text{H}\alpha}$  direction. The symbols indicate the variation in characteristic luminosity ( $L_{\text{H}\alpha}^*$ ) with respect to  $M_r$  for Saunders functional fits (blue) and Gaussian fits (green) to the  $\text{H}\alpha$  LFs in different  $M_r$  ranges. For the Saunders fits, the faint-end slope ( $\alpha$ ) is fixed at  $-1.16$ , the best-fitting  $\alpha$  of the  $z < 0.1$  GAMA  $\text{H}\alpha$  LF (Paper I). The horizontal errors indicate the  $M_r$  range probed and the vertical errors indicate the errors associated with best-fitting  $L_{\text{H}\alpha}^*$  values. In both cases, the variation in  $L_{\text{H}\alpha}^*$  can be approximated through a power law (equation 10).

To determine the best-fitting parameters for the higher- $z$  ( $0.1 \lesssim z \lesssim 0.34$ ) bivariate functions, we fix  $\alpha_M$  and  $\alpha_L$  to be equal to the best-fitting  $z < 0.1$  model values. The best-fitting values determined in this manner are given in Table 2. Given the lognormal like shape of the  $z < 0.1$  normalized bivariate function at brighter magnitudes (Fig. 11), which is similar to that exhibited by size–magnitude or colour–magnitude distributions (Choloniewski 1985; Chapman et al. 2003; Huang et al. 2013), a Schechter–Gaussian model can also be fitted,

$$\begin{aligned} \Psi(M_r, L_{\text{H}\alpha}) &= 0.4 \ln(10) \\ &\times 10^{-0.4(\alpha_M+1)(M_r-M_r^*)} \\ &\times \exp\left[-10^{-0.4(M_r-M_r^*)}\right] \\ &\times \exp\left\{-\frac{1}{2\sigma^2} \log^2\left[1 + \left(\frac{L_{\text{H}\alpha}}{L_{\text{H}\alpha}^*}\right) \left(10^{0.4\beta(M_r-M_r^*)}\right)\right]\right\}. \end{aligned} \quad (12)$$

The number density contours produced by a Schechter–Gaussian model is identical to the  $\Phi$  contours shown in Fig. 12. Therefore in Table 2 we also provide the best-fitting Schechter–Gaussian model parameters for the bivariate functions shown in Fig. 6. We note that in the two highest redshift bins, we have a limited sampling of galaxies fainter than broad-band or emission-line characteristic luminosity (Fig. 6).

More complex fitting methods like that of de Jong & Lacey (2000) could be considered, but for the purposes of this analysis, however, we find the best-fitting model shown in Fig. 12 to be sufficient as it provides a good qualitative and a quantitative description



**Figure 12.** The  $z < 0.1$  bivariate function and the fitted model (equation 11). The model contours provide a reasonable description of the data (black lines). The top and right-hand panels compare the univariate  $M_r$  (Fig. 8) and  $\text{H}\alpha$  (fig. 7 of Paper I) LFs (symbols) computed from integrating the  $z < 0.1$  bivariate  $M_r$ – $\text{H}\alpha$  function (Fig. 6 a) with that predicted by the model. Also shown are the Schechter and Saunders functional fits to the univariate  $M_r$  and  $\text{H}\alpha$  LFs (fig. 11 of Paper I). The colour scale indicates log number densities in the unit of  $\text{Mpc}^{-3} \text{dex}^{-1} \text{mag}^{-1}$ .

of the low- $z$  bivariate  $L_{\text{H}\alpha}$ – $M_r$  function. Furthermore, the approximate Schechter and Saunders functional fit forms of GAMA  $M_r$  and  $\text{H}\alpha$  LFs can be recovered from numerically integrating  $\Psi(M_r, L_{\text{H}\alpha})$  along  $L_{\text{H}\alpha}$  and  $M_r$  axes, respectively. Moreover, by integrating  $\Psi(M_r, L_{\text{H}\alpha})$  with respect to both  $L_{\text{H}\alpha}$  and  $M_r$ , we recover the  $z < 0.1$   $\text{H}\alpha$  SFRD reported in Paper I.

The best-fitting  $z < 0.1$   $M_r^*$  and  $\alpha_M$  bivariate model parameters given in Table 2 agree within uncertainty with the best-fitting parameters determined for the  $z < 0.1$  univariate  $M_r$  LF (Table 1). The relationship between  $\log L_{\text{H}\alpha}^*$  and  $M_r^*$  is emphasized in Fig. 11. Note that  $\alpha_L$  represents the faint-end slope, and the positive slope implies a decrease in number density. This is not unexpected as the distribution of the normalized number densities with respect to  $\log L_{\text{H}\alpha}$  within a given magnitude bin has a lognormal shape than a power-law shape (Fig. 11).

In summary, both Schechter–Saunders and Schechter–Gaussian models are able to provide a good representation of the lowest- $z$  bivariate  $L_{\text{H}\alpha}$ – $M_r$  function. As for the higher- $z$  ( $0.1 \lesssim z \lesssim 0.34$ ) bivariate  $L_{\text{H}\alpha}$ – $M_r$  functions, the models provide a good description of the bright end of the bivariate functions, where the data exist. It becomes increasingly difficult to constrain the models to obtain a good description of the faint end of the bivariate functions with increasing redshift as the range in  $L_{\text{H}\alpha}$  and  $M_r$  probe decreases.

Finally, an alternative method of modelling bivariate functions is proposed by Takeuchi (2010). This approach relies on using a copula to connect two marginal distributions (e.g.  $\text{H}\alpha$  and  $M_r$  LFs) thereby constructing their bivariate distribution. Takeuchi (2010) and Takeuchi et al. (2013) advocate the use of a

**Table 2.** The best-fitting parameters of the fitted models (equation 11) to the four bivariate  $L_{\text{H}\alpha}$ – $M_r$  functions. The faint-end slopes,  $\alpha_M$  and  $\alpha_L$ , of the higher- $z$  bivariate models are fixed to be equal to the  $z < 0.1$  model values.

Parameter	Schechter–Saunders function			
	$0.001 < z < 0.1$	$0.1 < z < 0.15$	$0.17 < z < 0.24$	$0.24 < z < 0.34$
$M_r^*$	$-21.12 \pm 0.10$	$-21.57 \pm 0.09$	$-21.77 \pm 0.26$	$-22.62 \pm 0.14$
$\alpha_M$	$-1.29 \pm 0.06$	–	–	–
$\log \Psi^* (\text{Mpc}^{-3})$	$-4.17 \pm 1.54$	$-4.49 \pm 0.45$	$-4.79 \pm 0.49$	$-4.40 \pm 0.93$
$\log L_{\text{H}\alpha}^* (\text{W})$	$32.30 \pm 0.65$	$32.52 \pm 0.33$	$32.75 \pm 0.59$	$33.55 \pm 0.64$
$\beta$	$0.92 \pm 0.03$	$0.85 \pm 0.12$	$0.59 \pm 0.30$	$0.46 \pm 0.25$
$\sigma$	$0.46 \pm 0.03$	$0.46 \pm 0.05$	$0.48 \pm 0.04$	$0.38 \pm 0.10$
$\alpha_L$	$1.53 \pm 1.15$	–	–	–
Parameter	Schechter–Gaussian function			
	$0.001 < z < 0.1$	$0.1 < z < 0.15$	$0.17 < z < 0.24$	$0.24 < z < 0.34$
$M_r^*$	$-21.12 \pm 0.11$	$-21.57 \pm 0.10$	$-21.77 \pm 0.27$	$-22.63 \pm 0.14$
$\alpha_M$	$-1.29 \pm 0.06$	–	–	–
$\log \Psi^* (\text{Mpc}^{-3})$	$-2.67 \pm 0.14$	$-2.94 \pm 0.14$	$-3.08 \pm 0.33$	$-3.53 \pm 0.31$
$\log L_{\text{H}\alpha}^* (\text{W})$	$33.55 \pm 0.04$	$33.79 \pm 0.14$	$34.12 \pm 0.42$	$34.45 \pm 0.36$
$\beta$	$0.92 \pm 0.03$	$0.85 \pm 0.12$	$0.59 \pm 0.30$	$0.44 \pm 0.24$
$\sigma$	$0.45 \pm 0.02$	$0.45 \pm 0.04$	$0.47 \pm 0.04$	$0.38 \pm 0.07$

**Table 3.** The SFRDs estimated from integrating the best-fitting Schechter–Saunders models over three different  $L_{\text{H}\alpha}$  and  $M_r$  ranges. The limiting  $\log L_{\text{H}\alpha} = 37$  (denoted  $L_b$  in the table) and  $M_r = -26$  (denoted  $M_b$  in the table). The second column ( $\log \dot{\rho}_{*(0,0)-(L_b,M_b)}$ ) reports the SFRDs computed from integrating the best-fitting higher- $z$  bivariate models from  $[L_{\text{H}\alpha}, M_r] = [0, 0]$  to  $[L_b, M_b]$ , the third provides those computed from integrating the bivariate models between the observed lowest  $z < 0.1$   $\log L_{\text{H}\alpha}$  and  $M_r$  values and  $[L_b, M_b]$ , and the final column provides the SFRDs obtained from integrating each model between the lowest  $\log L_{\text{H}\alpha}$  and  $M_r$  values at their respective redshift range and  $[L_b, M_b]$ .

Redshift range	$\log \dot{\rho}_{*(0,0)-(L_b,M_b)} (\text{M}_{\odot} \text{yr}^{-1} \text{Mpc}^{-3})$	$\log \dot{\rho}_{*(L_{f,z1},M_{f,z1})-(L_b,M_b)} (\text{M}_{\odot} \text{yr}^{-1} \text{Mpc}^{-3})$	$\log \dot{\rho}_{*(L_{f,zx},M_{f,zx})-(L_b,M_b)} (\text{M}_{\odot} \text{yr}^{-1} \text{Mpc}^{-3})$
$0.1 < z < 0.15$	–2.00	–2.00	–2.03 (–14.25, 33.1)
$0.17 < z < 0.24$	–1.75	–1.77	–2.03 (–18.75, 33.5)
$0.24 < z < 0.34$	–1.72	–1.77	–2.14 (–19.25, 33.9)

Farlie–Gumbel–Morgenstern or a Gaussian copula, both of which are explicitly related to the linear correlation coefficient, to connect two given marginals. See Takeuchi (2010) and Sato, Ichiki & Takeuchi (2011) for a rigorous mathematical definition of copula theory, dependence measures between two variables and how to estimate the bivariate distribution given two or more marginals.

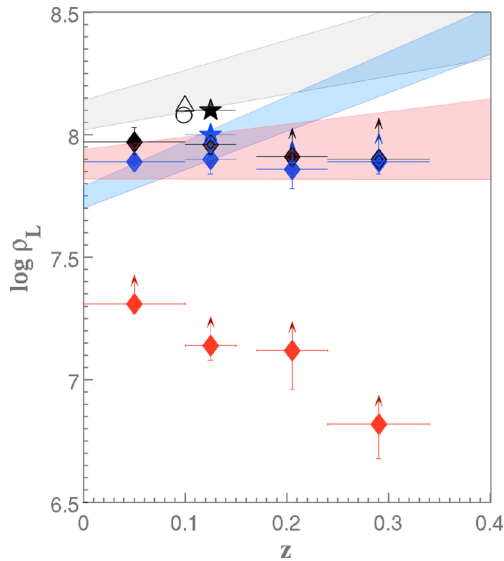
## 5.2 Luminosity and density of SF galaxies

In this section we present the  $r$ -band luminosity and  $\text{H}\alpha$  SFRD of SF galaxies computed from integrating the bivariate  $L_{\text{H}\alpha}$ – $M_r$  functions. The LD ( $\rho_L$ ) evolution observed in the GAMA  $\text{H}\alpha$  SF population is shown in Fig. 13. The filled diamonds indicate the GAMA  $\rho_L$  derived from integrating the four Schechter functions shown in Fig. 10. The filled stars indicate the density estimated from integrating the best-fitting Schechter–Saunders model to the  $0.1 < z < 0.15$  bivariate  $L_{\text{H}\alpha}$ – $M_r$  function. The  $z < 0.1$  estimate is not shown here as it is similar to that obtained from integrating the Schechter functional fit to the univariate  $z < 0.1$   $M_r$  LF. We also show the GAMA  $\rho_L$  confidence limits from Loveday et al. (2012), and low-redshift density estimates from Blanton et al. (2003a) and Montero-Dorta & Prada (2009). We see a result here that is consistent with the SF populations of the broad-band blue and red LFs shown in Fig. B2. That is the  $\rho_L$  estimated from the best-fitting Schechter functions do not show an evolution in  $\rho_L$  with redshift,

in contrast to Loveday et al. (2012). This is a natural consequence of the SF populations comprising only a small fraction (10–20 per cent) of the total red galaxy population. The decrease in  $\rho_L$  of blue SF population at higher- $z$  ( $z > 0.17$ ) is likely due to the difficulty in measuring  $\text{H}\alpha$  in higher- $z$  galaxy spectra as higher- $z$  galaxies are likely to have fainter optical magnitudes and lower spectral signal-to-noise ratio than their low- $z$  counterparts. Given that our sample is already biased against red SF galaxies, mainly as a result of the  $\text{H}\alpha$  flux limit, the drop in  $\rho_L$  corresponding those galaxies is not unexpected.

## 5.3 Star formation rate density

As mentioned previously, our primary motivation behind modelling the bivariate  $L_{\text{H}\alpha}$ – $M_r$  functions is to overcome the bivariate sample effects introduced by the dual  $\text{H}\alpha$  flux and magnitude selection imposed on our sample. As a result of this effect, the higher- $z$  ( $z > 0.1$ ) SFRDs presented in Paper I are underestimates. In Fig. 14 we show the SFRDs derived by integrating the bivariate analytic fit to  $L_{\text{H}\alpha}$ – $M_r$  function. The SFRDs derived this way are given in Table 3. By modelling the low-redshift  $L_{\text{H}\alpha}$ – $M_r$  distribution over  $-26 < M_r < -10$  and  $30 < \log L_{\text{H}\alpha} < 37$  (Fig. 12), and assuming the faint-end bivariate distribution is similar for the higher- $z$  ( $0.1 \lesssim z \lesssim 0.34$ ) samples, we can estimate a correction for the missing optically faint SF galaxies at those redshifts. In Fig. 14 it can



**Figure 13.** The evolution of  $r$ -band LD ( $\rho_L$ ) of all, and blue and red  $H\alpha$  SF galaxies as a function of redshift (filled black, blue and red symbols, respectively), compared to the evolution of  $\rho_L$  of all, and blue and red GAMA galaxies as a function of redshift (shaded regions; Loveday et al. 2012). Also shown are the  $\rho_L$  measurements at low- $z$  from Blanton et al. (2003a) (open circle) and Montero-Dorta & Prada (2009) (open triangle). The filled stars at  $z \sim 0.12$  indicate the value obtained from integrating the best-fitting bivariate analytical form for the  $0.1 < z < 0.15$  bivariate  $L_{H\alpha}-M_r$  function.

be seen that the resulting SFRDs are much more consistent with that from other published measurements (e.g. Hopkins & Beacom 2006), than the direct estimates from Paper I. Note that the two highest redshift ranges probed likely overestimated due to poorly constrained bivariate functions.

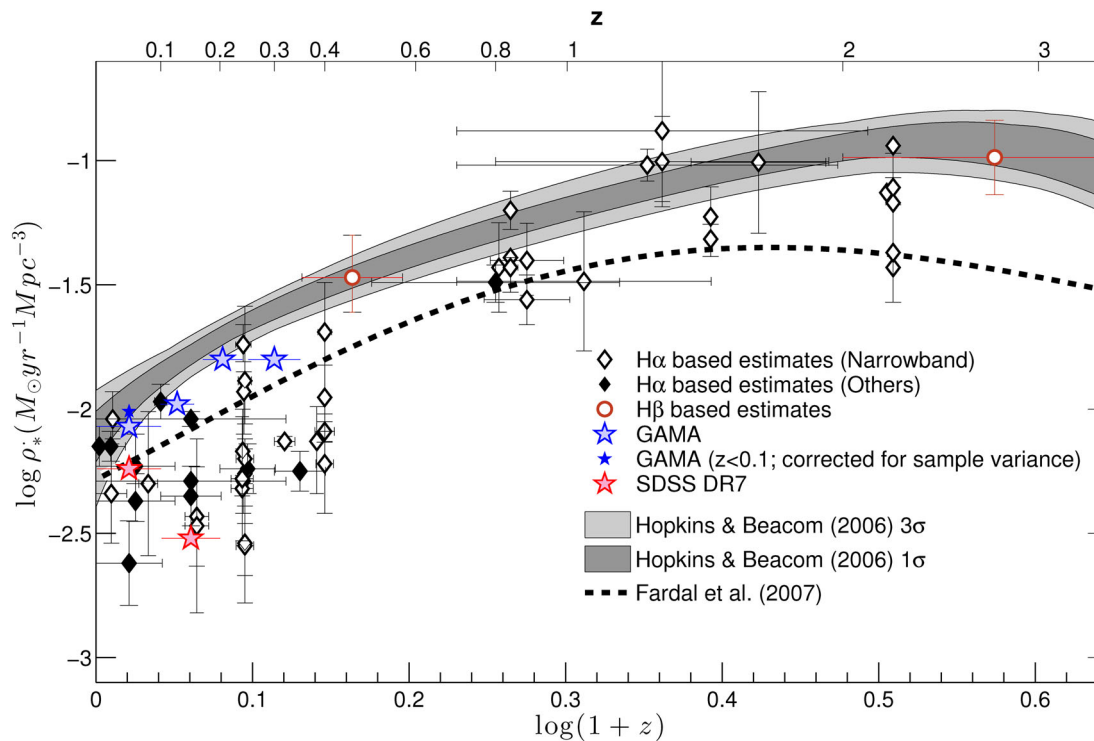
## 6 IMPORTANCE OF BIVARIATE $L_{H\alpha}-\mathcal{M}$ FUNCTIONS

In this section, we investigate the evolution of the bivariate  $L_{H\alpha}-\mathcal{M}$  functions, and the SMFs that result from integrating the bivariate functions for the full sample and those split by colour. The bivariate  $L_{H\alpha}-\mathcal{M}$  functions of all SF galaxies in redshift bins are shown Fig. 16. Again the lowest redshift function probes the largest range in both  $L_{H\alpha}$  and  $\mathcal{M}$ , while the higher- $z$  ( $0.1 < z < 0.34$ ) functions indicate a decrease in range with increasing redshift like seen in Fig. 6 for bivariate  $L_{H\alpha}-M_r$  functions.

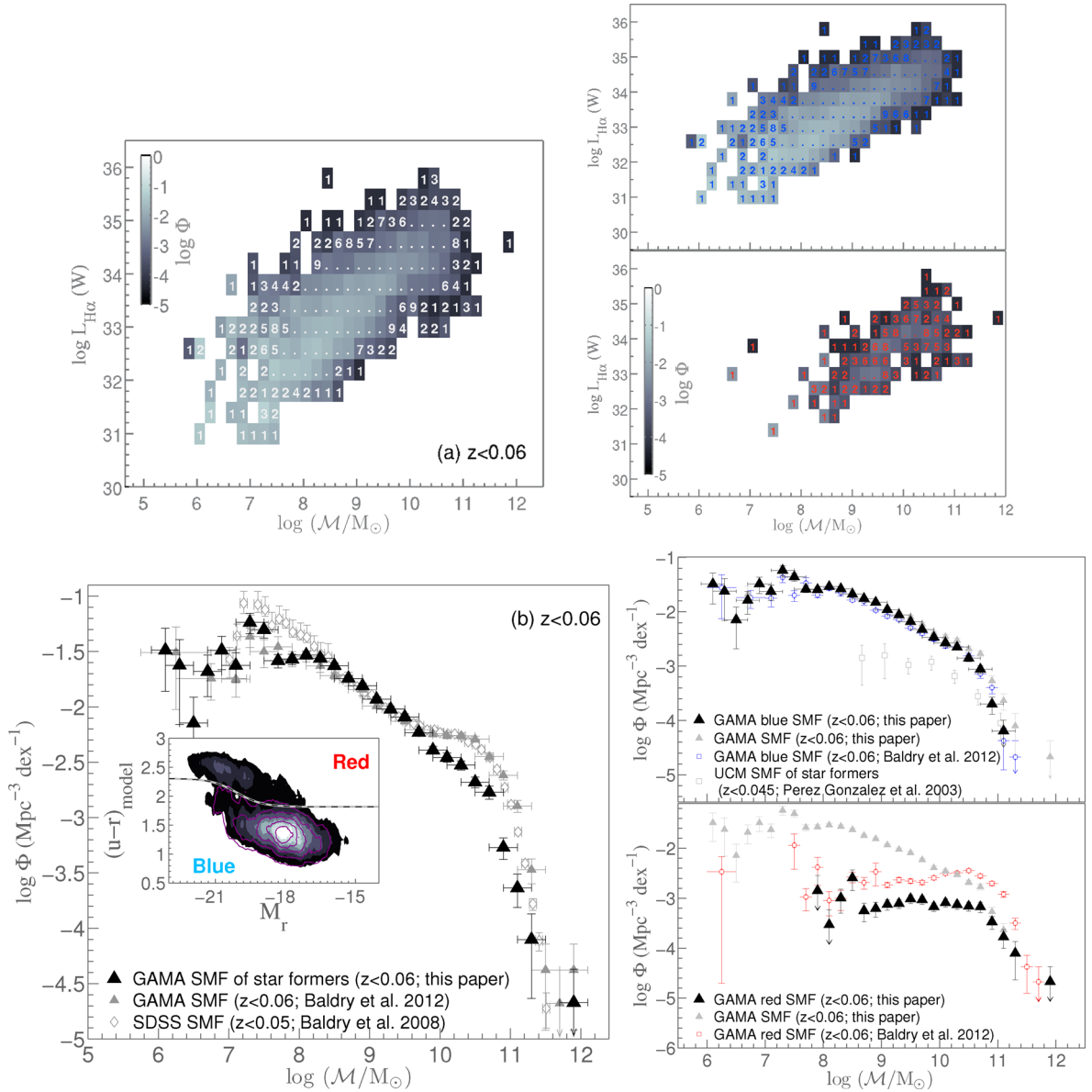
The colour–magnitude relation used to obtain a photometric blue and red classification in this part of the analysis is different to that discussed in Section 4. To reiterate, our objective in constructing bivariate and univariate SMFs of SF galaxies with blue and red photometric class is to compare our results with the existing GAMA results in a consistent manner (e.g. Baldry et al. 2012). For the stellar mass based analyses presented in the subsequent sections, as they are focused on SMFs, we use the colour–magnitude relation introduced by Baldry et al. (2004) (equation 11 of their paper) and used by Baldry et al. (2012) to construct the  $z < 0.06$  GAMA SMFs split by colour.

### 6.1 The stellar mass functions of SF galaxies

Baldry et al. (2012) present the  $z < 0.06$  GAMA galaxy SMFs determined using the density corrected  $1/V_{\max}$  method. They further



**Figure 14.** The cosmic SFR history, taken from Paper I, with our new measurements shown as blue stars. The GAMA SFRDs are based on integrating the analytic fits to the bivariate  $L_{H\alpha}-M_r$  LF (Figs 12). Published estimates based on narrow-band surveys and slitless spectroscopy data are shown as open symbols, and those based on broad-band surveys as filled symbols. The grey bands and the dashed line correspond to the best-fitting star formation histories of Hopkins & Beacom (2006) and Fardal et al. (2007), respectively.



**Figure 15.** (a) The  $z < 0.06$  GAMA bivariate  $L_{H\alpha}$ – $\mathcal{M}$  functions of all, blue and red SF galaxies (left, top right and bottom right panel, respectively), constructed using the density corrected  $1/V_{\text{max}}$  method described in Section 3.2. The grey-scale indicates the log of number densities ( $\Phi$ ) in the unit of  $\text{Mpc}^{-3} \text{dex}^{-2}$ . (b) The SMFs obtained from integrating the bivariate  $L_{H\alpha}$ – $\mathcal{M}$  functions over  $L_{H\alpha}$  are compared to the  $z < 0.06$  GAMA SMFs of Baldry et al. (2012) and  $z < 0.05$  SDSS SMF of Baldry, Glazebrook & Driver (2008).

show the GAMA SMFs of photometrically classified blue and red galaxies based on the colour  $(u-r)$ –magnitude ( $M_r$ ) relationship given in Baldry et al. (2004). Using the density corrected  $1/V_{\text{max}}$  method discussed in Section 3.2, we construct the  $z < 0.06$  bivariate  $L_{H\alpha}$ – $\mathcal{M}$  functions and the SMFs of  $H\alpha$  SF galaxies. Our results compared to the GAMA SMFs from Baldry et al. (2012) are shown in Fig. 15.

Fig. 15(a) left- and right-hand panels show the bivariate  $L_{H\alpha}$ – $\mathcal{M}$  functions of all SF galaxies (Fig. 15 a left), and photometrically classified blue and red SF subpopulations (Fig. 15 a right top and bottom panels, respectively). The bivariate  $L_{H\alpha}$ – $\mathcal{M}$  function of all  $z < 0.06$  SF galaxies comprise both blue and red galaxies, though dominated by blue ones. The SMFs obtained from integrating the bivariate functions over  $L_{H\alpha}$  are shown in Fig. 15(b), and the GAMA SMFs of Baldry et al. (2012) are also shown for comparison. Baldry

et al. (2012) find that the SMF of all  $z < 0.06$  GAMA galaxies is well described by a double Schechter function. This functional form has also been used by other authors (e.g. Popesso et al. 2006; Baldry et al. 2008; Drory et al. 2009; Pozzetti et al. 2010) to describe LFs and SMFs, and its origin is related to the bimodal colour–magnitude distribution of (blue and red) galaxies. Pozzetti et al. (2010) and Baldry et al. (2008) find that the massive end of the SMF ( $\log \mathcal{M} > 10.5 M_{\odot}$ ) is largely dominated by red galaxies while blue galaxies mainly contribute to the faint end of the SMF. This is also evident in the left-hand panel of Fig. 15(b) where the faint end of the SMF of all  $z < 0.06$  SF galaxies is well matched to that of the GAMA SMF (Baldry et al. 2012), while the bright end of the SMF shows a significant discrepancy. This disagreement arises naturally from our sample consisting only of  $H\alpha$  SF galaxies. As only a small fraction of photometrically classified  $z < 0.06$  red



galaxies have reliably detected H $\alpha$  emission, the GAMA SMF of  $z < 0.06$  SF galaxies (left-hand panel of Fig. 15 b) disagree with the SMF of Baldry et al. (2012) at the high-mass end. In the right top panel of Fig. 15, the SMF of blue galaxies (Baldry et al. 2012, open blue squares) and blue SF galaxies (black triangles) are in good agreement for most stellar masses. This indicates that the blue galaxies identified by the colour split advocated by Baldry et al. (2012) are mostly SF, with  $F_{\text{H}\alpha} > 1 \times 10^{-18} \text{ W m}^{-2}$ . The SMF of all SF galaxies (grey filled symbols) shows higher  $\Phi$  values at higher masses than the GAMA blue SMF as a result of the contribution from the photometrically classified red SF galaxies. The SMF of red SF galaxies is shown in the right bottom panel of Fig. 15(b). Within uncertainties the shape of the red SF SMF is similar to that of the red SMF indicating that red SF galaxies are a small and stellar mass independent fraction of all red galaxies. Fig. 15(b) (left-hand panel) inset shows the distribution of the H $\alpha$  SF sample (purple contours) compared to all  $z < 0.06$  galaxies regardless of star formation, and the dashed line is the colour–magnitude cut used here. Even though photometrically classified blue and red galaxy subpopulations are conventionally labelled as SF and passive galaxies, respectively, a sample selected on the basis of detected H $\alpha$  emission, which is a direct tracer of ongoing star formation in a galaxy, includes both photometrically classified blue and red galaxies. Furthermore, not all galaxies with a photometric blue classification have detected H $\alpha$  emission. The fact that some red galaxies are SF while some blue ones are not currently forming stars raises the question: does the shape of the SMF of blue and red SF galaxies has any dependence on the fraction of red star formers and blue non-star formers? The evidence of the existence red H $\alpha$  SF galaxies is more pronounced in the colour–magnitude distributions shown in Fig. 18, where the number statistics of red SF systems are higher than the  $z < 0.06$  sample.

Finally, it is interesting to note that approximately 20–30 per cent of photometrically classified red galaxies contributing to the GAMA red SMF of Baldry et al. (2012) at all stellar masses are SF. Approximately 40 per cent of the  $z < 0.06$  red SF galaxies in our sample are also detected at 250  $\mu\text{m}$  in Herschel–ATLAS survey, and they cover over two orders of magnitude in 250  $\mu\text{m}$  luminosity. Given dust is a requirement to be detected at this wavelength, it is reasonable to conclude that at least 40 per cent of the red star formers in our low-redshift sample are dusty SF galaxies and that dust in these galaxies contributes to their redder colour. Moreover, these dusty systems indicate a large spread in  $NUV - r$  colour, which is an indicator of recent star formation. This implies that in addition to the dusty ongoing star formation these galaxies are also dominated by old stars, and these old stars likely also contribute to the redder galaxy colour.

## 6.2 The stellar mass function of star-forming galaxies across time

We divide our sample into four redshift bins to investigate the evolution of the bivariate  $L_{\text{H}\alpha} - \mathcal{M}$  functions. For each redshift range, we construct the all H $\alpha$  SF (Fig. 16), and the respective photometrically classified blue and red SF bivariate functions (Fig. 17). The lowest redshift bivariate  $L_{\text{H}\alpha} - \mathcal{M}$  functions (i.e. all SF, and blue and red SF) probe the largest range in both  $L_{\text{H}\alpha}$  ( $30.5 \lesssim \log L_{\text{H}\alpha} \text{ W} \lesssim 36$ ) and  $\mathcal{M}$  ( $6 \lesssim \log \frac{\mathcal{M}}{M_{\odot}} \lesssim 12$ ), while the survey and sample selection effects dominate the higher- $z$  ( $0.1 \lesssim z \lesssim 0.34$ ) bivariate  $L_{\text{H}\alpha} - \mathcal{M}$  functions as evident from the decrease in  $L_{\text{H}\alpha}$  and  $\mathcal{M}$  ranges.

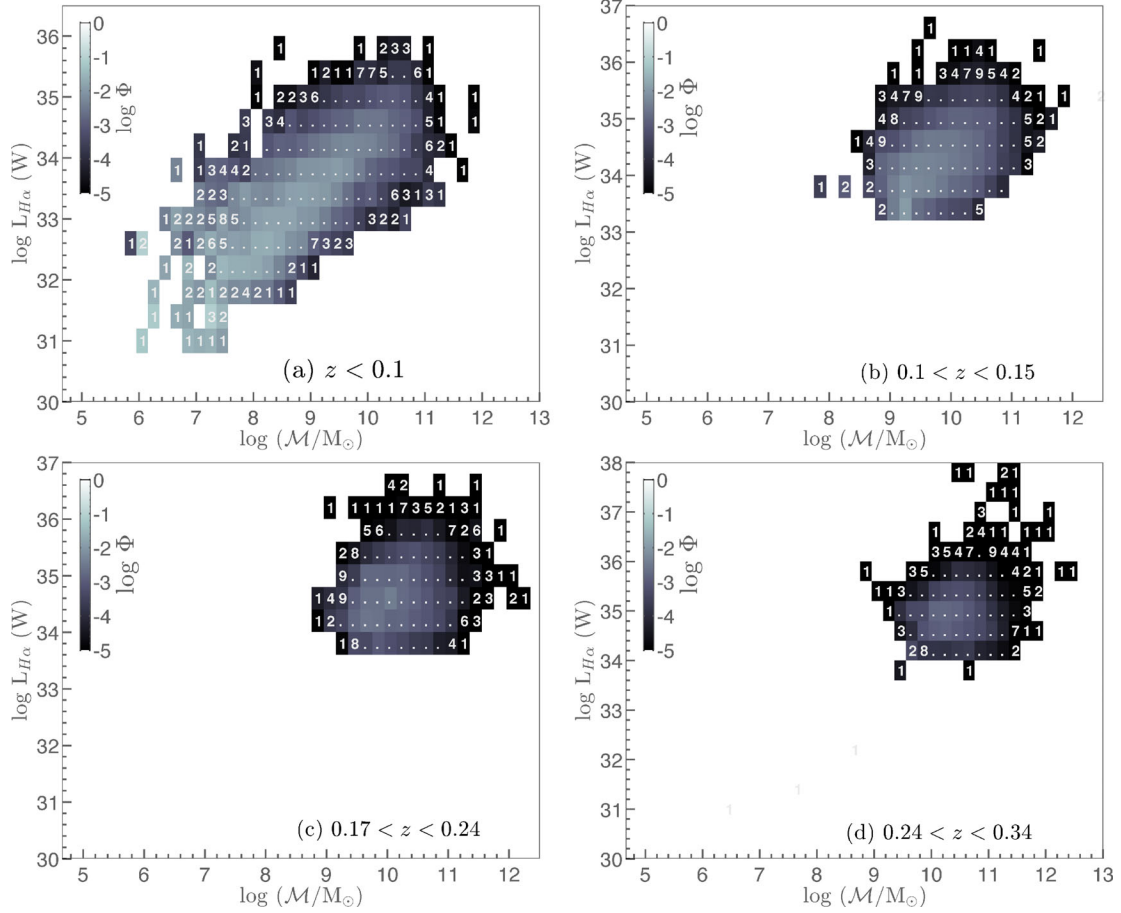
The SMFs computed from integrating each bivariate function over  $L_{\text{H}\alpha}$  are compared with the published measurements that cover similar redshift ranges. The results for the  $z < 0.1$  range are shown in Fig. 18, and Fig. 19 shows the higher- $z$  ( $0.1 \lesssim z \lesssim 0.34$ ) results. The  $z < 0.1$  SMF of all SF galaxies (the black filled symbols in all the panels of Fig. 18) is mostly in agreement, particularly at the low stellar mass end, with the photometrically classified blue SMFs of Baldry et al. (2004) and Moustakas et al. (2013). The differences seen at higher masses, where the  $z < 0.1$  blue SF SMF shows relatively lower amplitudes than other published measurements, highlight that not all photometrically classified blue sources are in fact SF. The same trend can be seen in the higher- $z$  SMFs shown in Fig. 19.

Overall, the trends discussed in Section 6.1 with regard to the  $z < 0.06$  SMF (i.e. the difference between the SMFs of all H $\alpha$  SF galaxies and all galaxies at the high-mass end that arises from the lack of many red SF galaxies, and that  $\sim 20$ –30 per cent of photometrically classified red galaxies contributing to the red SMFs are likely SF) are also evident in the higher- $z$  SMFs shown in Figs 18 and 19.

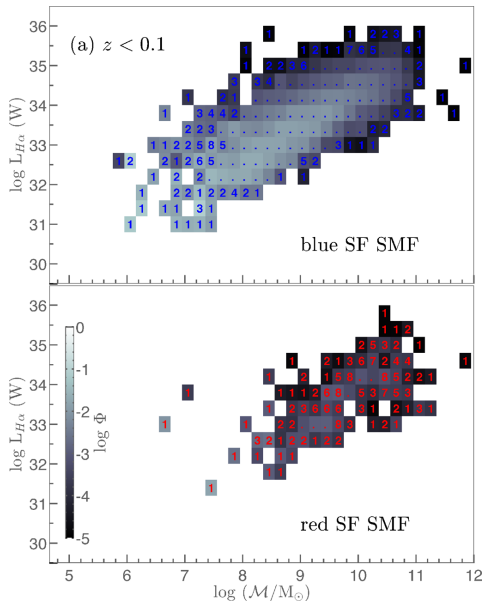
## 6.3 Mass-dependent evolution of the cosmic star formation history

To estimate an integrated stellar mass density ( $\rho^*$ ), we fit the analytic form introduced in Section 5 to the bivariate  $L_{\text{H}\alpha} - \mathcal{M}$  functions. We assume here that the relationship between  $\log L_{\text{H}\alpha}$  and  $M_r$  derived in Section 5 is also valid for  $\log L_{\text{H}\alpha}$  and  $\log \mathcal{M}$ , which is not an unreasonable assumption given the observed tight relationship between  $M_r$  and  $\log \mathcal{M}$ . In this interpretation of equation (11),  $\log \mathcal{M}^0$ , the equivalent of  $M_r^0$ , is fixed at  $\log \mathcal{M}/M_{\odot} = 9$ . The best-fitting functions for the  $z < 0.1$  and  $0.1 < z < 0.15$  bivariate  $L_{\text{H}\alpha} - \mathcal{M}$  functions are shown in the top panels of Fig. 20. Our model provides a good description of the observed low-redshift bivariate  $L_{\text{H}\alpha} - \mathcal{M}$  function, especially the faint-end distribution of the function. As the ranges in  $L_{\text{H}\alpha}$  and  $\mathcal{M}$  sampled by the observed  $0.1 < z < 0.15$  bivariate  $L_{\text{H}\alpha} - \mathcal{M}$  function are significantly less than that of the  $z < 0.1$  function, the two model parameters,  $\alpha_L$  and  $\alpha_M$ , that describe the faint-end shape a bivariate function are assumed to be equal to the best-fitting  $z < 0.1$  model values for the  $0.1 < z < 0.15$  bivariate function. The best-fitting parameters for the  $z < 0.1$  and  $0.1 < z < 0.15$  redshift bins are given in Table 4.

The relationship between  $\mathcal{M}$  and  $\rho^*$  for a fixed redshift range is shown in the bottom panel of Fig. 20. Overplotted in Fig. 20 are the  $\mathcal{M}$  and  $\rho^*$  relationships based on SDSS data at  $z \sim 0.05$  and ROLES data at  $z \sim 1$  (Li et al. 2011) and Sobral et al. (2014) HiZELS data at  $z \sim 0.4, 0.8, 1.47$  and  $2.23$ . The GAMA  $\mathcal{M}$  and  $\rho^*$  relationship agrees well with that of Li et al. (2011) and Sobral et al. (2014). Also, a comparison between GAMA, SDSS, ROLES and HiZELS results indicates that the shape of the relationship does not vary as a function of redshift, rather it is the normalization of the relationship that change. In the context of galaxy downsizing (Cowie, Songaila & Barger 1999), which states that high-mass galaxies formed their stars early and rapidly while low-mass counterparts formed stars at a slower rate and later times, the peak of the  $\log \mathcal{M}$  and  $\rho^*$  relationship is expected to shift towards lower masses with decreasing redshift. The lack of such a change contradicts the downsizing scenario, however, as Gilbank et al. (2010) point out high SFR galaxies are also likely to be high-mass systems that both dominate the galaxy numbers and  $\rho^*$  at high- $z$ , which could be considered ‘downsizing’.



**Figure 16.** The bivariate  $L_{H\alpha}$ – $\mathcal{M}$  functions of all SF galaxies split into four redshift bins. The grey-scale indicates the log number densities ( $\Phi$ ) in the unit of  $\text{Mpc}^{-3} \text{dex}^{-2}$ .



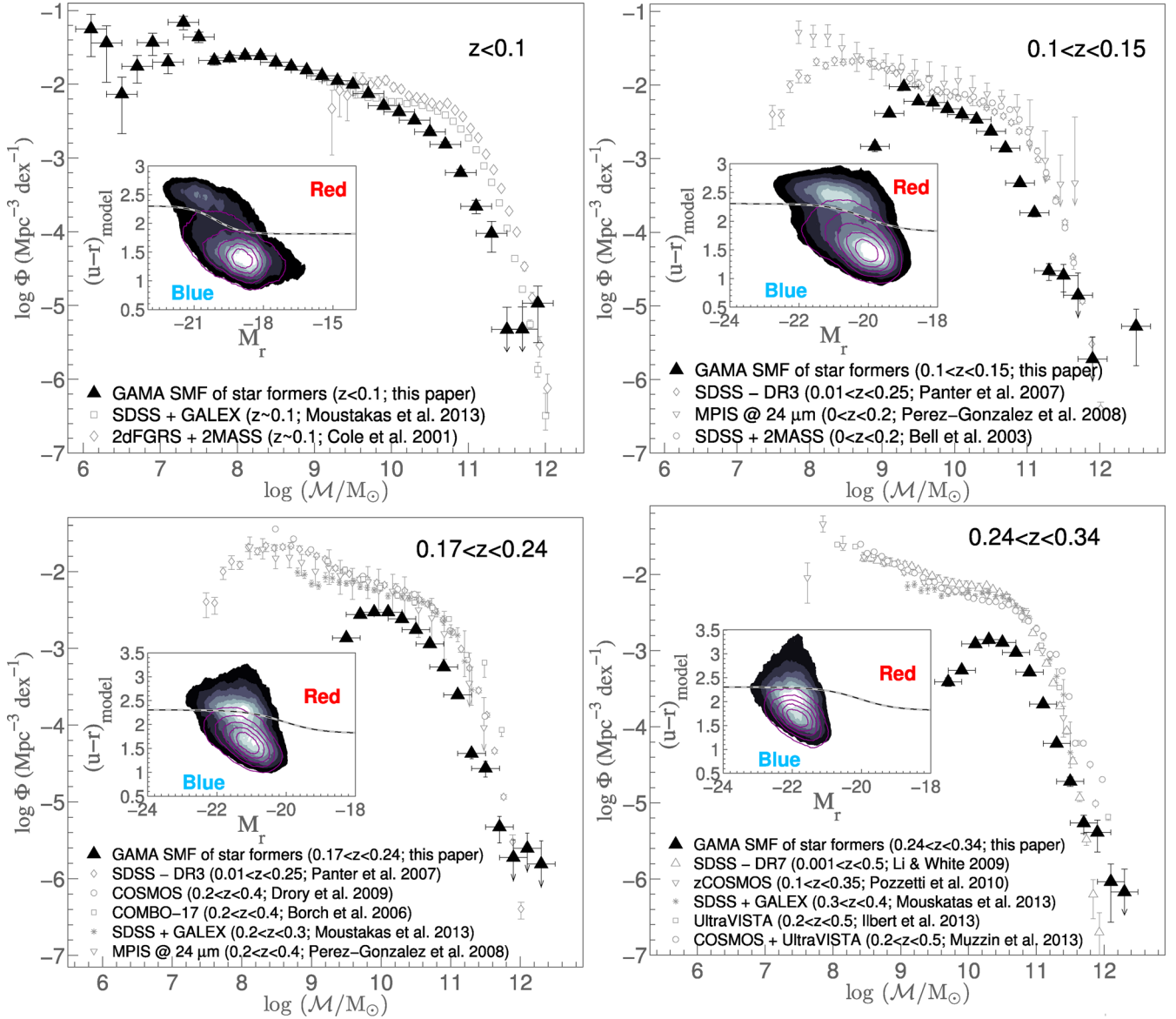
**Figure 17.** The  $z < 0.1$  bivariate  $L_{H\alpha}$ – $\mathcal{M}$  functions of blue and red sub-populations. The grey-scale indicates the log number densities ( $\Phi$ ) in the unit of  $\text{Mpc}^{-3} \text{dex}^{-2}$ , and is valid for both blue and red bivariate functions corresponding to the given redshift range.

The  $\rho^*$  derived from integrating the best-fitting analytic functions to the bivariate  $L_{H\alpha}$ – $\mathcal{M}$  functions (Fig. 20) are shown in Fig. 21. Also shown for comparison are the  $\rho^*$  measurements from recent studies at various redshifts up to  $z \sim 3$ . Note that all published measurements shown in Fig. 21 are calculated from univariate SMFs based on galaxy samples drawn from broad-band surveys. The data indicated as lower limits show the cases where the authors have provided a  $\rho^*$  measurement by integrating the univariate SMF down to a limiting  $\mathcal{M}$ , rather than to zero.

The left-hand panel of Fig. 21 compares the GAMA  $\rho^*$  measurements based on bivariate  $L_{H\alpha}$ – $\mathcal{M}$  functions with those derived from total galaxy samples, and the right-hand panel of the same figure compares our results with the  $\rho^*$  based on photometrically classified blue galaxy populations and emission-line samples. Our results agree well with the published measurements based on photometrically classified blue galaxy populations and emission-line samples, as might be expected given the close agreement between our SF SMFs and the blue galaxy SMF of Baldry et al. (2012).

## 7 DISCUSSION

In this followup paper to Paper I, we have explored the bivariate  $L_{H\alpha}$ – $\mathcal{M}_r$  and  $L_{H\alpha}$ – $\mathcal{M}$  functions of GAMA galaxies. One of the main aims of this analysis is to investigate whether we can reliably recover a correction for the incompleteness introduced in selecting  $H\alpha$  detected objects from an  $r$ -band limited survey by modelling the low-redshift bivariate function, which can then be used as a



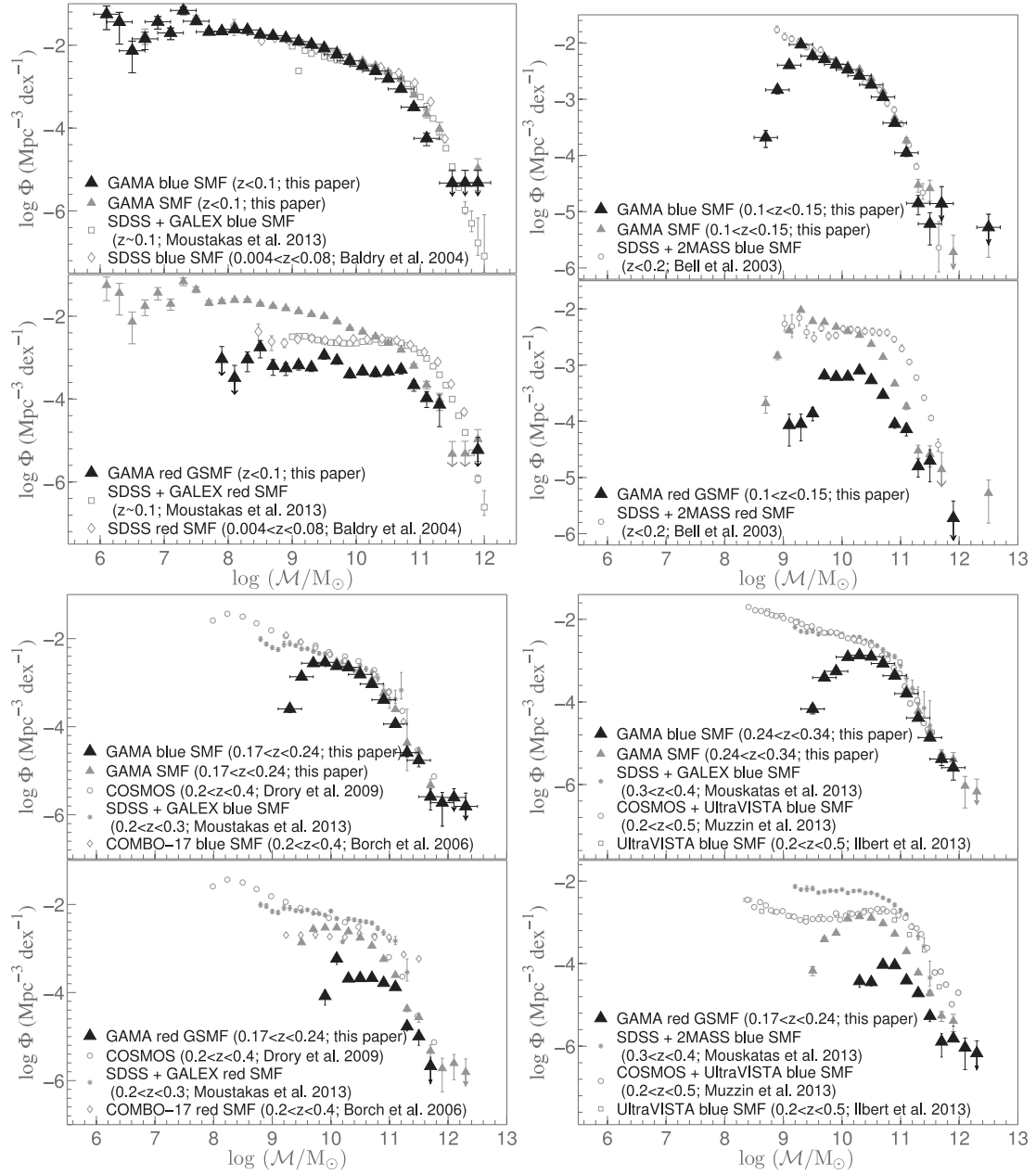
**Figure 18.** The SMFs derived from integrating the  $z > 0.1$   $L_{\text{H}\alpha}$ – $\mathcal{M}$  functions shown in Fig. 16 over  $L_{\text{H}\alpha}$  for all SF galaxies. The redshift increases from top to bottom. All published SMF measurements shown are adjusted to our assumed cosmology and to a Chabrier (2003) IMF.

reference to account for the missing optically faint SF galaxies at higher- $z$  ( $0.1 \lesssim z \lesssim 0.34$ ). Other goals of this investigation include exploring the evolution of bivariate and univariate functions of all  $\text{H}\alpha$  SF and photometrically classified blue and red  $\text{H}\alpha$  SF subpopulations relative to the evolution of all, blue and red populations regardless of star formation, and the mass dependence of the SFR history.

In order to compare our results more directly with the previous GAMA LF (Loveday et al. 2012) and SMF (Baldry et al. 2012) studies, we adopt the LF estimators used in their studies to construct the bivariate functions. A discussion on the formulation of three LF estimators (e.g. the classical method, density corrected  $1/V_{\text{max}}$  method and SWML estimator) for constructing bivariate functions is presented in Section 3 and in Appendix A. A comparison between the bivariate functions based on these methods (Section 3.3) shows that the differences in number densities are largely limited to the faint  $L_{\text{H}\alpha}$ – $M_r$  end of the bivariate functions as expected given the relatively small volumes sampled. The bivariate  $L_{\text{H}\alpha}$ – $M_r$  functions

and LFs presented in Section 4 and  $L_{\text{H}\alpha}$ – $\mathcal{M}$  functions and SMFs presented in Section 6.2 are based on the classical  $1/V_{\text{max}}$  method, and are compared with the GAMA LFs of Loveday et al. (2012) and other published measurements that are mostly based on the classical method. The bivariate  $L_{\text{H}\alpha}$ – $\mathcal{M}$  functions and SMFs presented in Section 6.1 are based on the density corrected  $1/V_{\text{max}}$ , and are compared with the GAMA SMFs (Baldry et al. 2012) based on the same method.

As a consequence of the magnitude-limited nature of the GAMA survey, the bivariate  $L_{\text{H}\alpha}$ – $M_r$  functions (Fig. 6 and 7) show a progressive decline in number density ( $\Phi$ ) towards fainter  $L_{\text{H}\alpha}$  and  $M_r$ , and the range in  $L_{\text{H}\alpha}$  and  $M_r$  probed decrease with increasing redshift. In each of the four redshift bins considered, the  $L_{\text{H}\alpha}$ – $M_r$  range, and the  $\Phi$  values of the blue  $\text{H}\alpha$  SF bivariate functions (Fig. 7) are similar to that of the total  $\text{H}\alpha$  SF functions (Fig. 6). This result indicates that the galaxies contributing to the total  $\text{H}\alpha$  SF bivariate functions are mostly drawn from the photometrically classified blue subpopulations at each redshift. This is further collaborated



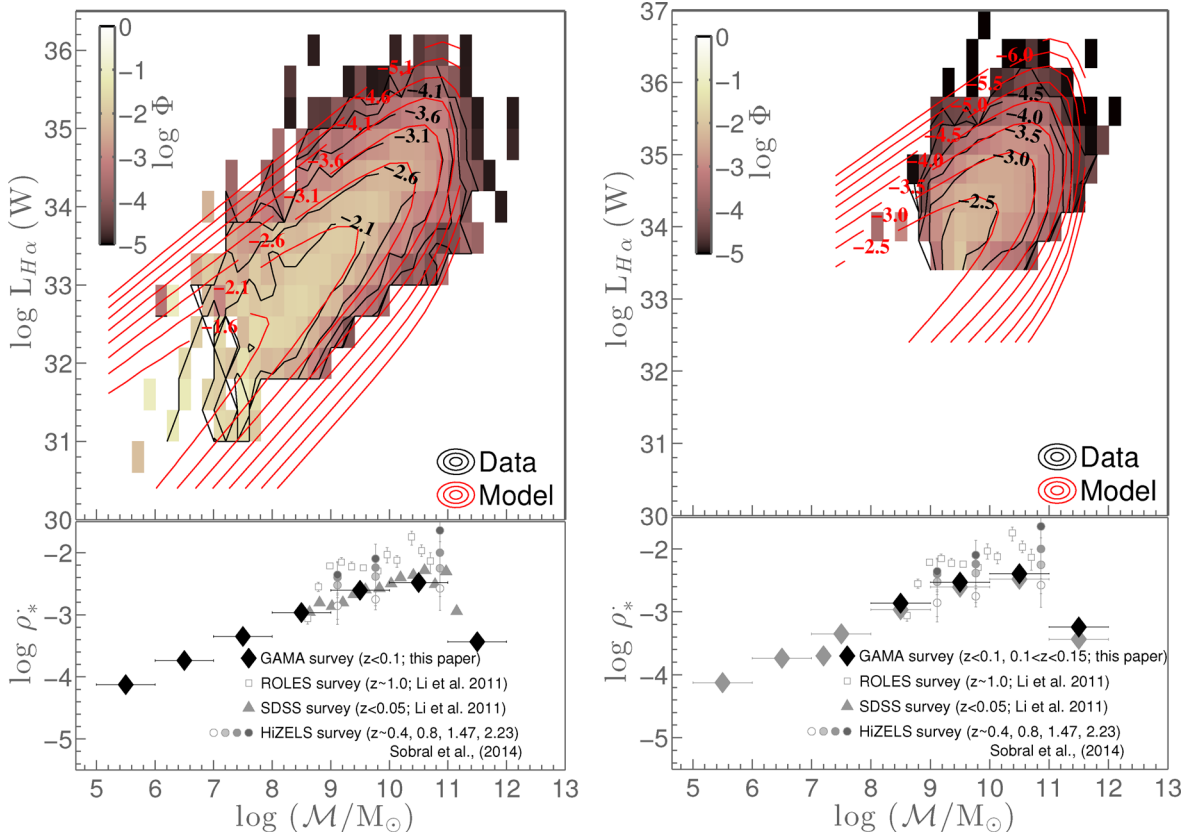
**Figure 19.** The blue (first panel in each two panel set) and red (second panel in each two panel set) SMFs derived from integrating the blue and red bivariate functions shown in Figs 17 and B1. All published SMF measurements shown are adjusted to our assumed cosmology and to a Chabrier (2003) IMF.

by both the relatively smaller range in  $L_{\text{H}\alpha}$  and  $M_r$  probed by the red bivariate  $L_{\text{H}\alpha}$ – $M_r$  functions (Fig. B2), and the close agreement between the Loveday et al. (2012) blue LFs and those computed by integrating the blue bivariate  $L_{\text{H}\alpha}$ – $M_r$  functions at all redshifts (Figs 8 and B2). While the red  $\text{H}\alpha$  SF bivariate number density contribution to the total  $\text{H}\alpha$  SF bivariate function is relatively low, Figs 8 and B2 show that a fraction of galaxies classified as red at all  $M_r$  values are in fact SF galaxies. The same conclusion is drawn from the bivariate  $L_{\text{H}\alpha}$ – $\mathcal{M}$  functions (Figs 15, 16 and 17), and the SMFs (Figs 18, 19) computed from integrating the bivariate functions. While the  $\text{H}\alpha$  SF galaxy population at each redshift is primarily drawn from the blue subpopulations at that redshift, approximately 20–30 per cent of those photometrically classified as red and conventionally called passive are forming massive stars at all stellar

masses at each redshift range probed. We find that  $\sim 40$  per cent of the red  $\text{H}\alpha$  SF galaxies at  $z < 0.06$  (i.e. those contributing to the red bivariate  $L_{\text{H}\alpha}$ – $\mathcal{M}$  function and the SMF shown in Fig. 15) is also reliably detected at  $250\,\mu\text{m}$ . As dust is a requirement to be able to be detected at this wavelength, it is likely that some of the red  $\text{H}\alpha$  SF galaxies are dusty (and therefore red) SF systems. Moreover, those detected at  $250\,\mu\text{m}$  cover a large range in  $NUV - r$  colour, which is an indicator of recent star formation in galaxies, and most lie below the  $NUV - r = 5.4$  cutoff for recent star formation from Schawinski et al. (2007). This points to red  $\text{H}\alpha$  SF galaxies having a non-negligible fraction of underlying old stellar population that likely also contribute to their redder colour.

Motivated by the analytic formalism widely used to model bivariate brightness profiles (e.g. de Jong & Lacey 2000; Cross





**Figure 20.** Top panels: best-fitting analytic models (red lines) to the  $z < 0.1$  (left) and  $0.1 < z < 0.15$  (right) bivariate  $L_{H\alpha}$ – $\mathcal{M}$  functions. The colour scale and black contours indicate the log number densities ( $\Phi$ ) in the unit of  $\text{Mpc}^{-3} \text{dex}^{-2}$ . Bottom panels: SFRD ( $\text{M}_{\odot} \text{yr}^{-1} \text{Mpc}^{-3} \text{dex}^{-1}$ ) versus stellar mass for the  $z < 0.1$  (left) and  $0.1 < z < 0.15$  (right). For reference, we show the Li et al. (2011) results based on SDSS (filled triangles) and ROLES (open squares) surveys and the Sobral et al. (2014) results based on HiZELS (circles) survey. All measurements are adjusted to the Baldry & Glazebrook (2003) IMF.

**Table 4.** The best-fitting Schechter–Saunders parameters for the  $z < 0.1$  and  $0.1 < z < 0.15$  bivariate  $L_{H\alpha}$ – $\mathcal{M}$  functions.

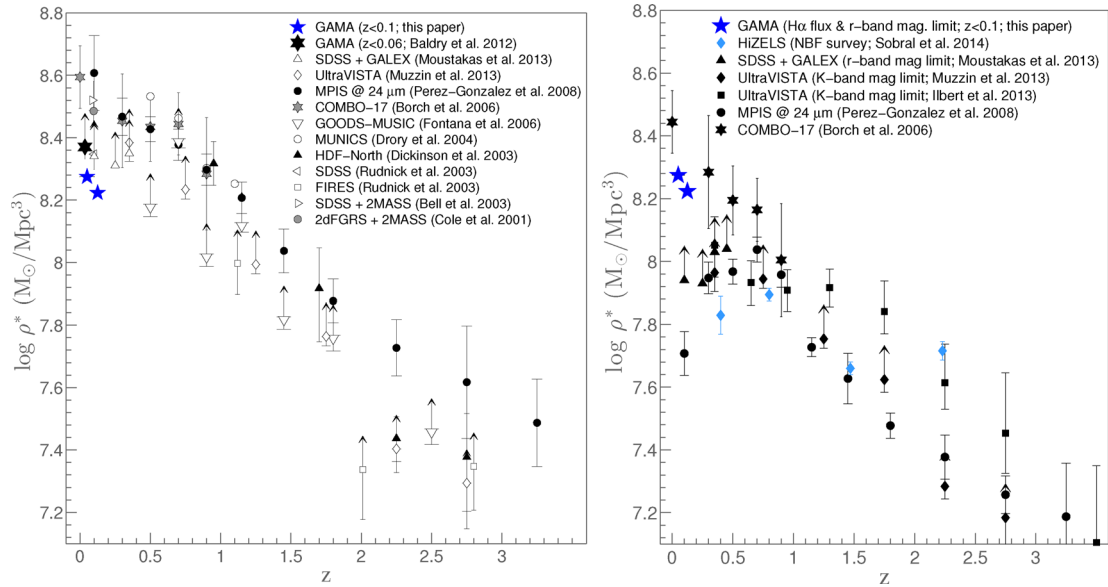
Parameter	$z < 0.1$	$0.1 < z < 0.15$
$\log \mathcal{M}^*/\text{M}_{\odot}$	$10.77 \pm 0.07$	$10.84 \pm 0.04$
$\alpha_{\mathcal{M}}$	$-1.31 \pm 0.04$	–
$\log \Psi^* (\text{Mpc}^{-3})$	$-4.50 \pm 1.23$	$-4.77 \pm 0.20$
$\log L_{H\alpha}^* (\text{W})$	$32.00 \pm 0.58$	$32.06 \pm 0.20$
$\beta$	$0.70 \pm 0.02$	$0.67 \pm 0.05$
$\sigma$	$0.50 \pm 0.02$	$0.52 \pm 0.03$
$\alpha_L$	$1.20 \pm 0.93$	–

et al. 2001; Driver et al. 2005; Ball et al. 2006), in Section 5.1 we introduce a simple analytic model to describe the observed bivariate functions presented in Sections 4 and 6. This model assumes that the bivariate function can be written as a product of two functions (Choloniewski 1985; Corbelli et al. 1991), a Schechter (1976) function representative of  $r$ -band LFs (or SMFs) and a Saunders et al. (1990) function representative of  $H\alpha$  LFs (Paper I). The two functional forms are linked in the bivariate analytic relation through the observed relationship between  $L_{H\alpha}^*$  and  $M_r$  (Fig. 11). The resultant best-fitting bivariate models to the  $z < 0.1$   $L_{H\alpha}$ – $M_r$ , and the  $z < 0.1$  and  $0.1 < z < 0.15$   $L_{H\alpha}$ – $\mathcal{M}$  functions (Figs 12 and 20, respectively) provide a good description of the data, and from integrating the best-fitting  $z < 0.1$  bivariate  $L_{H\alpha}$ – $M_r$  model we were able to recover the  $z < 0.1$  SFRD reported in Paper I. The same model is

used to fit the rest of the higher- $z$  bivariate functions by fixing the faint  $L_{H\alpha}$ – $M_r$  (or  $\mathcal{M}$ ) end slope of the bivariate model to be equal to that of the lowest redshift best-fitting model. As the  $0.1 < z < 0.15$  bivariate functions sample a relatively large range in both  $L_{H\alpha}$  and  $M_r$  (or  $\mathcal{M}$ ), the  $0.1 < z < 0.15$  models can be reasonably well constrained, however, the two higher- $z$  models cannot be constrained accurately even with assuming a constant faint-end slope (i.e. assuming no evolution in the faint-end slope of the bivariate function) for the model. This is mainly due to the observed higher- $z$  bivariate  $L_{H\alpha}$ – $M_r$  and  $L_{H\alpha}$ – $\mathcal{M}$  functions being incomplete at around the characteristic  $L_{H\alpha}$ ,  $M_r$  and  $\mathcal{M}$  values. By integrating the best-fitting higher- $z$  bivariate  $L_{H\alpha}$ – $M_r$  and  $L_{H\alpha}$ – $\mathcal{M}$  models, particularly the  $0.1 < z < 0.15$  models which we were able to constrain more accurately than the rest, we were able to recover an approximate correction for the missing optically faint SF galaxies. The SFRDs estimated this way (Fig. 14) show a strong evolution with redshift up to  $z \sim 0.2$  and a flattening thereafter, though, we caution against using the last two points (i.e. the two higher- $z$   $\rho^*$  measurements) in understanding the evolution of  $\rho^*$  with redshift as their best-fitting bivariate models are not well constrained.

The mass dependence of the SFR history and the evolution of the stellar mass density of  $H\alpha$  SF galaxies are explored in Section 6.3. The  $\mathcal{M}$  and  $\rho^*$  relationship for the  $z < 0.1$  and  $0.1 < z < 0.15$  redshift ranges derived from integrating the respective GAMA bivariate  $L_{H\alpha}$ – $\mathcal{M}$  functions is shown in Fig. 20. Also shown for comparison are the  $\mathcal{M}$ – $\rho^*$  relations derived using SDSS data at  $z \sim 0.05$ , ROLES data at  $z \sim 1$  (Li et al. 2011) and HiZELS data at  $z \sim 0.4$ ,





**Figure 21.** The evolution of the  $\rho^*$ , as traced by all galaxies (left) and blue/emission-line galaxies (right). GAMA measurements are derived from integrating the analytic forms shown in the top panels of Fig. 20. In the right-hand panels photometrically classified blue galaxies are shown with black symbols, while coloured symbols refer to emission-line samples. The cases where the authors report a density value evaluated assuming a lower mass limit are shown as lower limits. All measurements are adjusted for our assumed cosmology and for a Chabrier (2003) IMF.

0.8, 1.47, 2.23 (Sobral et al. 2014). A comparison between GAMA, SDSS, ROLES and HiZELS  $\mathcal{M}-\rho^*$  results show that while the normalization of the  $\mathcal{M}-\rho^*$  relation increases modestly with redshift, the shape remains the same. This result points towards a scenario where SFRs in all galaxies, regardless of stellar mass, decline with decreasing redshift (Karim et al. 2011; Sobral et al. 2014), contradicting the ‘galaxy downsizing’ scenario of Cowie et al. (1999), which states that massive galaxies formed their stars early at a rapid rate while the low-mass systems formed stars late at a slower rate. However, as Gilbank et al. (2010) point out the high SFR systems are also likely to be high-mass galaxies that dominate SFRD at high- $z$ , which could be considered as downsizing.

Finally, the evolution of the  $\rho^*$  of H $\alpha$  SF population in comparison to the evolution of  $\rho^*$  of all galaxies regardless of star formation, and of photometrically classified blue galaxies (black markers) and other H $\alpha$  galaxy samples (coloured markers) is shown in the left and right-hand panels of Fig. 21, respectively. The SMDs based on the H $\alpha$  SF sub sample of GAMA galaxies lie low in the left-hand panel of Fig. 21. This is expected as other stellar mass densities shown in that panel are based on all galaxies regardless of star formation. Our results are in good agreement with the SMDs based only on either the blue subpopulation of galaxies or other emission-line samples (right-hand panel of Fig. 21). The scatter in data points is most likely due to cosmic (sample) variance.

## 8 SUMMARY

We have explored the GAMA bivariate  $L_{\text{H}\alpha}-M_r$  and  $L_{\text{H}\alpha}-\mathcal{M}$  functions in the paper, and the key results of the analysis are as follows.

(i) By modelling the low-redshift distribution of the bivariate  $L_{\text{H}\alpha}-M_r$  function, we estimate a correction for the missing optically faint SF galaxies at higher- $z$ . The corrected stellar mass densities presented in Fig. 14 show high level of consistency with earlier published results (e.g. Hopkins & Beacom 2006), suggesting that this approach is reasonable. The implication is that the shape of the

faint end of the bivariate functions does not evolve strongly over this redshift range.

(ii) The H $\alpha$  SF sample used for this study consists of both photometrically classified blue and red galaxies, though dominated by blue galaxies. This allows us to construct not only the bivariate and univariate LFs and SMFs of H $\alpha$  SF galaxies, but also the LFs and SMFs of photometrically classified blue and red SF galaxies.

(iii) The  $M_r$  LFs obtained from integrating the bivariate  $L_{\text{H}\alpha}-M_r$  functions are in agreement with the GAMA  $M_r$  LFs of photometrically classified blue galaxies from Loveday et al. (2012). Also, the  $z < 0.06$  SMF of SF galaxies obtained from integrating the bivariate  $L_{\text{H}\alpha}-\mathcal{M}$  function is consistent with the blue SMF of Baldry et al. (2012).

(iv) The low redshift ( $z < 0.15$ )  $\rho^*$  and  $\mathcal{M}$  relationship derived using GAMA data agree well with the  $z < 0.05$  results from Li et al. (2011). The shape of the  $\rho^*$  and  $\mathcal{M}$  relationship at low redshift is similar to that obtained at  $z \sim 1$  from Gilbank et al. (2010). The comparison of results between different redshifts indicate that the shape of the  $\rho^*$  and  $\mathcal{M}$  relationship does not change with redshift over the redshift range probed by the bivariate functions, rather it is the normalization of the relationship at all masses that varies with redshift.

(v) The GAMA stellar mass densities based on bivariate  $L_{\text{H}\alpha}-\mathcal{M}$  functions, i.e. based on a sample of H $\alpha$  SF galaxies, is in good agreement with the published stellar mass densities based on photometrically classified blue galaxy populations.

## ACKNOWLEDGEMENTS

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GAMA is a joint European-Australasian project based around a spectroscopic campaign using the Anglo-Australian Telescope. The GAMA input catalogue is based on data taken from the Sloan Digital Sky Survey and the UKIRT Infrared Deep Sky Survey. Complementary imaging of the GAMA regions is being obtained by a number of independent survey programs including *GALEX* MIS, VST KIDS, VISTA VIKING, *WISE*, *Herschel*-ATLAS, GMRT and ASKAP providing UV to radio coverage. GAMA is funded by the STFC (UK), the ARC (Australia), the AAO, and the participating institutions. The GAMA website is <http://www.gama-survey.org/>.

Data used in this paper will be available through the GAMA website (<http://www.gama-survey.org/>) once the associated redshifts are publicly released.

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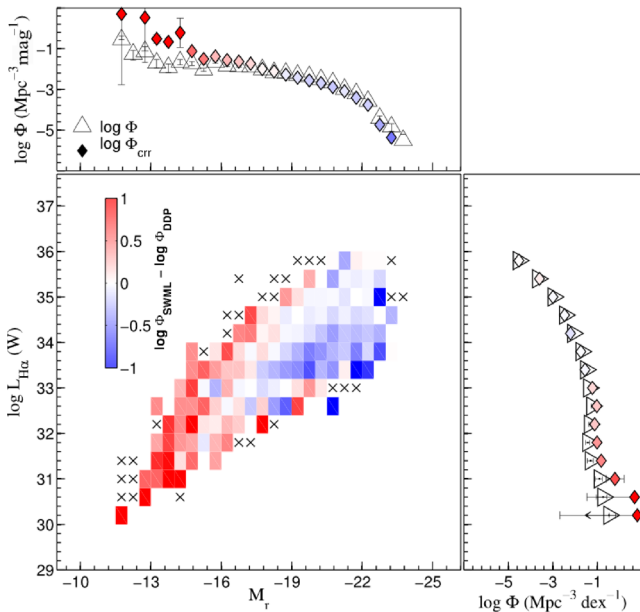
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## APPENDIX A: BIVARIATE STEPWISE MAXIMUM LIKELIHOOD METHOD

In order to overcome the inconvenience of not being able to adequately visualize whether the chosen parametrization represents a good fit to the data (Willmer 1997), a ‘non-parametric’ LF SWML LF estimator is introduced by Efstathiou et al. (1988). The SWML method does not suffer from the same biases that affect  $1/V_{\max}$  technique. Unlike  $1/V_{\max}$  method, the luminosity bins in SWML are highly correlated, so any issue that occurs affect the whole LF.

Efstathiou et al. (1988) describe the SWML formulation for univariate LFs. An extension of the SWML LF estimator for bivariate LFs is discussed in Sodre & Lahav (1993) for the bivariate diameter–



**Figure A1.** The residual map showing the difference in number densities calculated using the density corrected  $1/V_{\max}$  ( $\log \Phi_{\text{DDP}}$ ) and SWML ( $\log \Phi_{\text{SWML}}$ ) methods for the  $z < 0.1$  bivariate functions. The crosses indicate where the galaxy number statistics are low. Top and right-hand panels show the univariate LFs inferred from the bivariate functions.

LF and in Driver et al. (2005) for the bivariate brightness distribution. Following Efstathiou et al. (1988), Sodre & Lahav (1993) and Driver et al. (2005), we construct a bivariate SWML estimator for  $H\alpha/M_r$  and  $H\alpha$ /stellar mass LFs.

The probability of observing a galaxy  $i$  with a  $H\alpha$  luminosity  $L_i$  and an absolute  $r$ -band magnitude  $M_{r,i}$  at a redshift  $z_i$ , inclusive of the completeness/selection function ( $f$ ), is defined to be (Sodre & Lahav 1993; Loveday 2000; Ball et al. 2006)

$$p_i \propto \frac{\Phi(M_i, L_i) f(M_i, L_i, z_i)}{\int_{L_{\text{faint}}(z_i)}^{L_{\text{bright}}(z_i)=\infty} \int_{M_{\text{faint}}(z_i)}^{M_{\text{bright}}(z_i)} \Phi(M, L) f(M, L, z) dM dL}. \quad (\text{A1})$$

The probability,  $p_i$ , is directly proportional to the differential LF at  $M_i$  and  $L_i$ , and inversely proportional to the faintest and brightest absolute magnitudes ( $M_{\text{faint}}$ ,  $M_{\text{bright}}$ ) and  $H\alpha$  luminosities ( $L_{\text{faint}}$ ,  $L_{\text{bright}}$ ) visible at  $z_i$  (Heyl et al. 1997; Willmer 1997). Note that we do not account for survey incompleteness via  $f$ . Instead the incompleteness corrections defined in Paper I are incorporated into the bivariate LF through a weighting function (Driver et al. 2005) as explained below.

The SWML LF is derived by maximizing the likelihood function,  $\mathcal{L} = \prod_{i=1}^{N_g} p_i$ , generally  $\log \mathcal{L}$ , with respect to the discretized luminosity distribution. The bivariate LF,  $\Phi(M, L)$ , is parametrized as  $N_M$  and  $N_L$  steps (Efstathiou et al. 1988):

$$\Phi(M, L) = \Phi_{j,k} \quad j = 1, \dots, N_M, \quad \text{and} \quad k = 1, \dots, N_L, \quad (\text{A2})$$

where  $N_M$  and  $N_L$  are evenly spaced bins in  $M_r$  and  $H\alpha$  luminosity with  $M_j - \frac{\Delta M}{2} \leq M_i \leq M_j + \frac{\Delta M}{2}$ , and  $L_k - \frac{\Delta L}{2} \leq L_i \leq L_k + \frac{\Delta L}{2}$ .

Rewriting the denominator of equation (A1) in summation notation gives the following log-likelihood function

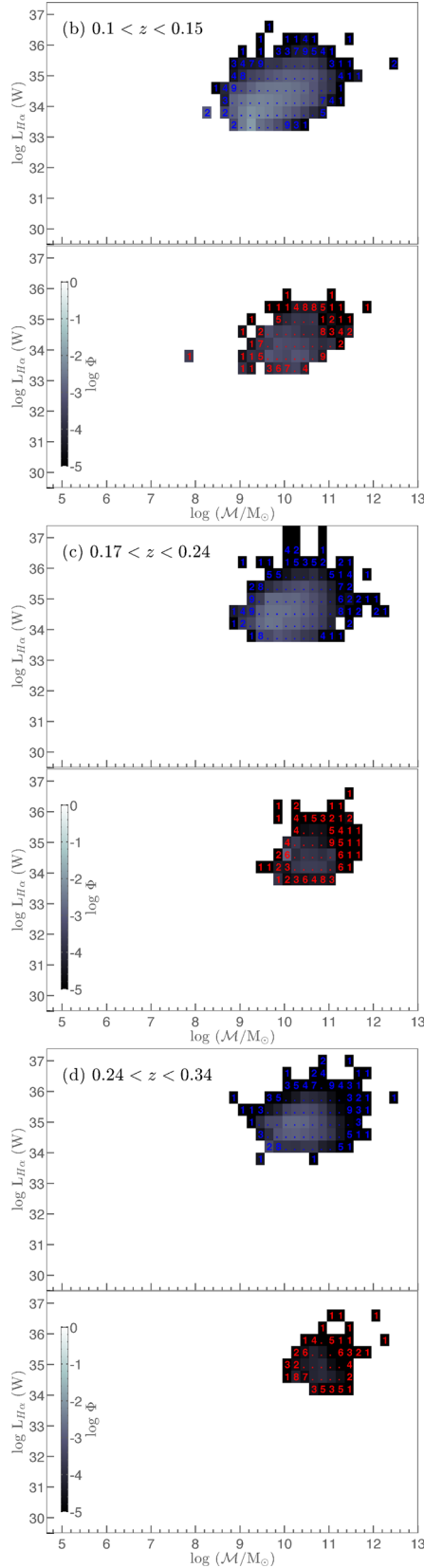
$$\begin{aligned} \ln \mathcal{L} = & \sum_{i=1}^{N_g} \sum_{j=1}^{N_M} \sum_{k=1}^{N_L} W[M_i - M_j, L_i - L_k] \ln \Phi_{jk} - \sum_{i=1}^{N_g} \ln \\ & \times \left\{ \sum_{a=1}^{N_M} \sum_{b=1}^{N_L} \Phi_{ab} \Delta M \Delta L H[M_a - M_{\text{faint}}(z_i), L_b - L_{\text{faint}}(z_i)] \right\} \\ & + C, \end{aligned} \quad (\text{A3})$$

where  $C$  is a constant,  $W[M_i - M_j, L_i - L_k]$  is the weighting function defined as (Driver et al. 2005)

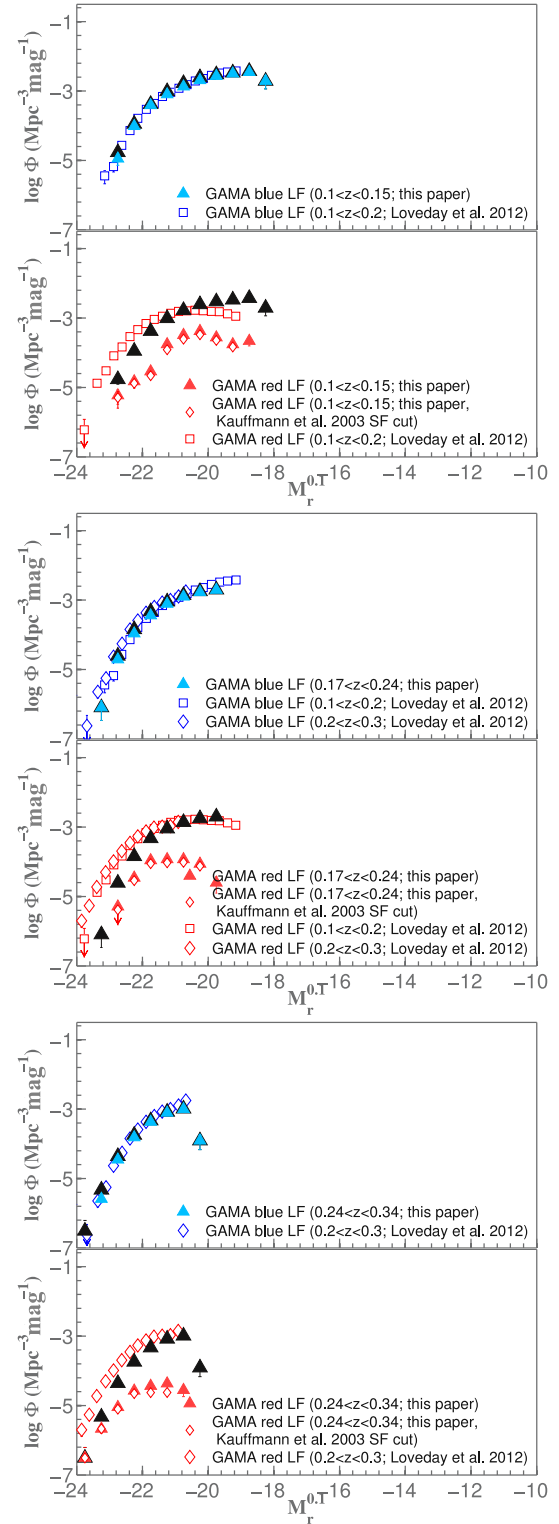
$$W[M_i - M_j, L_i - L_k] = \begin{cases} 1 & \text{if } M_j - \frac{\Delta M}{2} \leq M_i \leq M_j + \frac{\Delta M}{2} \\ & \text{and } L_k - \frac{\Delta L}{2} \leq L_i \leq L_k + \frac{\Delta L}{2} \\ 0 & \text{otherwise,} \end{cases} \quad (\text{A4})$$

and  $H[M_j - M_{\text{faint}}(z_i), L_k - L_{\text{faint}}(z_i)]$  is the ramp function, inclusive of incompleteness corrections ( $c$ ; Paper I), defined as

$$\begin{aligned} H[M_j - M_{\text{faint}}(z_i), L_k - L_{\text{faint}}(z_i)] \\ = \frac{1}{\Delta M \Delta L} \int_{M_j - \frac{\Delta M}{2}}^{M_j} dM \int_{L_k - \frac{\Delta L}{2}}^{L_k + \frac{\Delta L}{2}} dL O_i(M, L) c. \end{aligned} \quad (\text{A5})$$



**Figure B1.** The bivariate  $L_{H\alpha}$ – $M$  functions of blue and red subpopulations split into four redshift ranges. The grey-scale indicates the log number densities ( $\Phi$ ) in the unit of  $\text{Mpc}^{-3} \text{dex}^{-2}$ , and is valid for both blue and red bivariate functions corresponding to the given redshift range.



**Figure B2.** Each two sets of panels from top to bottom show the blue and red SF  $M_r$  functions derived from integrating the bivariate  $L_{H\alpha}$ – $M_r$  functions shown in Fig. 7 over  $L_{H\alpha}$ . The key indicate the redshift ranges probed. The all SF  $M_r$  functions are shown in Figs 8 and 9. The colour cut given in equation (4) used to classify galaxies as blue and red. The open red diamonds in each red SF  $M_r$  functions panels indicate the red SF  $M_r$  function constructed using the Kauffmann et al. (2003) SF/AGN prescription instead of that of Kewley & Dopita (2002).

The definitions of  $M'$  and  $L'$  are

$$M' = \max \left\{ M_j - \frac{\Delta M}{2}, \min \left[ M_j + \frac{\Delta M}{2}, M_{\text{faint}}(z_i) \right] \right\}$$

$$L' = \max \left\{ L_k - \frac{\Delta L}{2}, \min \left[ L_k + \frac{\Delta L}{2}, L_{\text{faint}}(z_i) \right] \right\}.$$

Simply setting  $M'$  or  $L'$  to be equal to the faint magnitude or luminosity bin boundary will result in the LF being underestimated in incompletely sampled bins. If the faint bin boundaries are used in the ramp function, then the incomplete bins should be excluded (Loveday et al. 2012). Finally, the function,  $O_i(M, L)$ , in equation (A5) is the observable window function for each galaxy at  $z_i$  (Driver et al. 2005), and has the following form.

$$O_i(M, L) = \begin{cases} 1 & \text{if } M_{\text{bright},i} \leq M_i \leq M_{\text{faint},i} \\ & \text{and } L_{\text{faint},i} \leq L_i \\ 0 & \text{otherwise,} \end{cases} \quad (\text{A6})$$

where,

$$M_{\text{bright},i} = m_{\text{bright}} - 5 \log d_{L(z_i)} - 25 - K(z_i)$$

$$M_{\text{faint},i} = m_{\text{faint}} - 5 \log d_{L(z_i)} - 25 - K(z_i)$$

$$L_{\text{faint},i} = 4\pi d_{L(z_i)}^2 f_{\text{H}\alpha, \text{faint}}$$

and

$$m_{\text{bright},i} = 14.65 \quad (\text{SDSS bright magnitude limit})$$

$$m_{\text{faint},i} = 19.4 \text{ or } 19.8 \quad (\text{GAMA faint magnitude limits})$$

$$f_{\text{faint},i} = 1 \times 10^{-18} \text{ W/m}^2 \quad (\text{H}\alpha \text{ flux limit; Paper I}).$$

As discussed in Efsthathiou et al. (1988) a constraint must be imposed on the likelihood to fix the normalization constant in equation (A3), by using a Lagrangian multiplier ( $\lambda$ ). We adopt the constraint used by Ball et al. (2006) and Sodre & Lahav (1993),

$$g = \sum_{j=1}^{N_M} \sum_{k=1}^{N_L} \Phi_{jk} \Delta M \Delta L - 1 = 0. \quad (\text{A7})$$

The likelihood with the constraint applied,  $\ln \mathcal{L}' = \ln \mathcal{L} + \lambda g(\Phi_{jk})$ , is maximized with respect to  $\Phi_{jk}$  and  $\lambda$ , requiring  $\lambda = 0$ . The constraint, although it does not affect the shape of the LF determined by  $\Phi_{jk}$ , plays a role in the error determination (Efsthathiou et al. 1988). The maximum likelihood (i.e.  $\partial \ln \mathcal{L}' / \partial \Phi_{jk} = 0$ ) is then given by

$$\Phi_{jk} \Delta M \Delta L = \frac{\sum_{i=1}^{N_g} W [M_i - M_j, L_i - L_k]}{\sum_{i=1}^{N_g} \left\{ \frac{H[M_j - M_{\text{faint}}(z_i), L_k - L_{\text{faint}}(z_i)]}{\sum_{a=1}^{N_M} \sum_{b=1}^{N_L} \Phi_{ab} \Delta M \Delta L H[M_a - M_{\text{faint}}(z_i), L_b - L_{\text{faint}}(z_i)]} \right\}}, \quad (\text{A8})$$

where  $\Phi_{ab}$  is from the previous iteration.

The bivariate SWML LF ( $\Phi_{jk}$ ), by construction, loses the information regarding the absolute normalization (Efsthathiou et al. 1988). We achieve a normalization for the bivariate SWML LF results presented in this paper by matching the bright end of the bivariate SWML LFs to their  $1/V_{\text{max}}$  LFs. While the shape of the faint-end  $1/V_{\text{max}}$  LF can be affected by the over/under densities, this estimator provides reliable abundances at higher luminosities, where it probes a relatively larger volume (Driver et al. 2005; Eke et al. 2005). Therefore, matching to the bright end of the LFs is a robust approach to fix the normalization of the bivariate SWML LF.

The LF errors can be determined using the fact that maximum likelihood estimates ( $\Phi_{jk}$ ) are asymptotically normally distributed with the covariance matrix:

$$\text{cov}(\Phi_{jk}) = I^{-1}(\Phi_{jk}), \quad (\text{A9})$$

where  $I$  is the information matrix:

$$I(\Phi_{jk}) = - \begin{pmatrix} \frac{\partial^2 \ln \mathcal{L}}{\partial \Phi_{ik} \partial \Phi_{jk}} + \frac{\partial g}{\partial \Phi_{ik}} \frac{\partial g}{\partial \Phi_{jk}} & \frac{\partial g}{\partial \Phi_{jk}} \\ \frac{\partial g}{\partial \Phi_{ik}} & 0 \end{pmatrix}.$$

The  $\text{H}\alpha$  and  $M_r$  LFs can be recovered from equation (A8) by summing over  $M_r$  and  $L_{\text{H}\alpha}$ , respectively. For example, to recover the  $\text{H}\alpha$  univariate LF,

$$\phi_{\text{H}\alpha} = \sum_{j=1}^{N_M} \phi_{jk} \times \Delta M. \quad (\text{A10})$$

Fig. A1 shows the residual between the bivariate LFs based on the density corrected  $V_{\text{max}}$  and bivariate SWML methods. Again the differences are primarily in the faintest bins of the lowest redshift sample. The differences here are greater than seen with the density corrected  $1/V_{\text{max}}$  method. Given the small number of galaxies contributing to these faintest bins, the discrepancy here should be interpreted primarily as a limit on the systematic uncertainty of the measurement of the faint end of the bivariate function, from all of these methods.

## APPENDIX B: BIVARIATE AND UNIVARIATE FUNCTIONS

The higher- $z$  bivariate  $L_{\text{H}\alpha} - \mathcal{M}$  functions and the univariate LF are shown in Figs B1 and B2.

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